Modified Schrödinger Equation for Particles with Mass of the Order of Human Neuron Mass

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Abstract
In this paper the modified Schrödinger equation (MSE) for the particles with mass = mass of the human neuron is obtained and solved. Considering that neuron mass is of the order of Planck mass it was shown that for mass of the order of the human neuron mass the transition quantum → classical behavior can occurs. Moreover it was argued that the human brain can be described as the fluid of the Planck particles. It is interesting to observe that the Planck gas was created at the beginning of the Universe.

Key Words: Modified Schrödinger Equation, Planck particles, neurons.

1. Introduction
Quantum theory describes the extraordinary behavior of the matter and energy which comprise our Universe at a fundamental level. At the root of quantum theory is the wave/particle duality of atoms, molecules and their constituting particles. A quantum system such as an atom or sub-atomic particle which remains isolated from its environment behaves as a wave of possibilities and exists in a coherent complex number valued “superposition” of many possible states. The behavior of such wave-like quantum level objects can be satisfactorily described in terms of state vector which evolves deterministically according to the Schrödinger equation (unitary evolution) denoted by U.

Somehow quantum microlevel superposition leads to unsuperposed stable structures in our macro-world. In a transition known as a wave function collapse or reduction (R), the quantum wave to alternative possibilities reduces to a simple macroscope reality, an “eigenstate” of some appropriate operator (Hameroff, 2003)

In seminal paper (Tarlaci, 2010) Tarlaci described the strong connection and mutual interaction of quantum theory and cognitive science. The master equation of quantum theory is the Schrödinger equation. In this paper we intend to biologize Schrödinger equation in order to find out the strongest relation between neuroscience and quantum physics.

In our paper (Pelc, 2010) we developed new form of Schrödinger equation. Modified Schrödinger Equation (MSE) with new term which describes the “memory” of the quantum state. In this paper we will show that memory of all quantum state is influenced by gravity.

As was shown in paper (Hameroff, 2003) the R states of quantum physics are originated at Planck level. In this paper we underline the fact that the Planck mass is of
the order of the mass of the human neuron, \( M_p = m_{HN} = 10^{-3} \) g. It is interesting to observe that Planck mass is the mass at which the modified Schrödinger equation changes the structure from parabolic for mass \( m < M_p = m_{HN} \) to hyperbolic for mass \( m > M_p = m_{HN} \).

1. Modified Schrödinger Equation

Quantum phenomena become increasingly important and the limit to when we may be able to confirm all quantum principles experimentally is still an open question. In the paper (Nairz, 2003) the discussion of fullerene (C_{60} molecule) experiments demonstrate the basic wave-particle duality for the most massive, most complex and most classical single object. The quantum behavior of the “particle” does not depend on the dimension of the particle (Kozłowski, 2005) it's depends on the realization of the circumstances, so that no information of the state of the object is collected before the measurement is performed (Kozłowski, 2005).

The quantum description of the microtubules (Kozłowski, 2005; Marcjak-Kozłowska, 2009; Kozłowski, 2009) and neurons was presented in papers (Mitra, 2006; Mitra, 2007) In this paper we develop the theoretical model for the study quantum processes in neurons. We argue that the equality of the neuron mass and Planck mass enables the applications of the equation developed in monograph (Kozłowski, 2006) to the investigation of the quantum processes in neurons in the context of thermal processes.

When Max Planck made the first quantum discovery he noted an interesting fact (Kozłowski, 2006). The speed of light, Newton’s gravity constant and Planck’s constant clearly reflect fundamental properties of the world. From them it is possible to derive the characteristic mass \( M_p \), length \( L_p \) and time \( T_p \) with approximate values

\[
L_p = 10^{-35} \text{ m}
\]
\[
T_p = 10^{-43} \text{ s}
\]
\[
M_p = 10^{-5} \text{ g}.
\]

Nowadays much of cosmology is concerned with “interface” of gravity and quantum mechanics.

Let us consider the question: how gravity can modify the quantum mechanics, i.e., the nonrelativistic Schrödinger equation (SE). We argue that SE with relaxation term describes properly the quantum behavior of particle with mass \( m_i < M_p \) and contains the part which can be interpreted as the pilot wave equation. For \( m_i \rightarrow M_p \) the solution of the SE represent the strings with mass \( M_p \).

The thermal history of the system (heated gas container, neuron Universe) can be described by the generalized Fourier equation (Kozłowski, 2006)

\[
q(t) = -\int_{-\infty}^{t} K(t-t') \vartheta T(t')dT'.
\] (1)

In Eq. (1) \( q(t) \) is the density of the energy flux, \( T \) is the temperature of the system and \( K(t-t') \) is the thermal memory of the system.

\[
K(t-t') = \frac{K}{\tau} \exp\left[-\frac{(t-t')}{\tau}\right],
\] (2)

where \( K \) is constant, and \( \tau \) denotes the relaxation time.

As was shown in (Kozłowski, 2006)

\[
K(t-t') = \begin{cases} K\delta(t-t') & \text{diffusion} \\ \frac{K}{\tau} \exp\left[-\frac{(t-t')}{\tau}\right] & \text{damped wave} \end{cases}
\]

The memory function \( K(t-t') \) describes the memory of the system, i.e. the system with diffusion memory lost its memory abruptly - by Dirac delta function. With \( K(t-t') = K = \text{constant} \), memory of the system is infinite. The \( K(t-t') \) which depends on the relaxation time \( \tau \) describes the medium with intermediate memory.

The damped wave or hyperbolic diffusion equation can be written as:

\[
\frac{\partial^2 T}{\partial t^2} + \frac{1}{\tau} \frac{\partial T}{\partial t} = \frac{D}{\tau} \nabla^2 T.
\] (3)

For \( \tau \rightarrow 0 \), Eq. (3) is the Fourier thermal equation
and $D_f$ is the thermal diffusion coefficient. The systems with very short relaxation time have very short memory. On the other hand for $\tau \to 0$ Eq. (3) has the form of the thermal wave (undamped) equation, or ballistic thermal equation. In the solid state physics the ballistic phonons or electrons are those for which $\tau \to \infty$. The experiments with ballistic phonons or electrons demonstrate the existence of the wave motion on the lattice scale or on the electron gas scale.

$$\frac{\partial^2 T}{\partial t^2} = \frac{D_f}{\tau} \nabla^2 T. \quad (5)$$

For the systems with very long memory Eq. (3) is time symmetric equation with no arrow of time, for the Eq. (5) does not change the shape when $t \to -t$.

In Eq. (3) we define:

$$\nu = \left( \frac{D_f}{\tau} \right), \quad (6)$$

velocity of thermal wave propagation and

$$\lambda = \nu \tau, \quad (7)$$

where $\lambda$ is the mean free path of the heat carriers. With formula (6) equation (3) can be written as

$$\frac{1}{\nu^2} \frac{\partial^2 T}{\partial t^2} + \frac{1}{\tau \nu^2} \frac{\partial T}{\partial t} = \nabla^2 T. \quad (8)$$

From the mathematical point of view equation:

$$\frac{1}{\nu^2} \frac{\partial^2 T}{\partial t^2} + \frac{1}{D} \frac{\partial T}{\partial t} = \nabla^2 T$$

is the hyperbolic partial differential equation (PDE). On the other hand Fourier equation

$$\frac{1}{D} \frac{\partial T}{\partial t} = \nabla^2 T \quad (9)$$

and Schrödinger equation

$$i \hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m_i} \nabla^2 \Psi \quad (10)$$

are the parabolic equations. Formally with substitutions

$$t \leftrightarrow it, \; \Psi \leftrightarrow T. \quad (11)$$

Fourier equation (9) can be written as

$$i \hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m_i} \nabla^2 \Psi \quad (12)$$

And by comparison with Schrödinger equation one obtains

$$D_f \hbar = \frac{\hbar^2}{2m_i} \quad (13)$$

$$D_f = \frac{\hbar}{2m_i}. \quad (14)$$

Considering that $D_f = \nu^2 \tau$ (6) we obtain from (14)

$$\tau = \frac{\hbar}{2m_i \nu \hbar}. \quad (15)$$

Formula (15) describes the relaxation time for quantum thermal processes. Starting with Schrödinger equation for particle with mass $m_i$ in potential $V$:

$$i \hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m_i} \nabla^2 \Psi + V \Psi \quad (16)$$

and performing the substitution (11) one obtains

$$\hbar \frac{\partial T}{\partial t} = \frac{\hbar^2}{2m_i} \nabla^2 T - VT \quad (17)$$

$$\frac{\partial T}{\partial t} = \frac{\hbar}{2m_i} \nabla^2 T - \frac{V}{\hbar} T. \quad (18)$$

Equation (18) is Fourier equation (parabolic PDE) for $\tau = 0$. For $\tau \neq 0$ we obtain
\[
\tau \frac{\partial^2 T}{\partial t^2} + \frac{\partial T}{\partial t} + \frac{V}{\hbar} T = \frac{\hbar}{2m_i} \nabla^2 T, \tag{19}
\]

\[
\tau = \frac{\hbar}{2m_i v^2} \tag{20}
\]
or

\[
\frac{1}{v^2} \frac{\partial^2 T}{\partial t^2} + \frac{2m_i}{\hbar} \frac{\partial T}{\partial t} + \frac{2V_{ri}}{\hbar} T = \nabla^2 T.
\]

With the substitution (11) equation (19) can be written as

\[
\frac{i\hbar}{\tau} \frac{\partial \Psi}{\partial \tau} = \nabla \Psi - \Psi - \frac{\hbar}{\tau} \frac{\partial^2 \Psi}{\partial \tau^2} \tag{21}
\]

The new term, relaxation term

\[
\tau \frac{\partial^2 \Psi}{\partial \tau^2} \tag{22}
\]
describes the interaction of the particle with mass \(m_i\) with space-time. When the quantum particle is moving through the quantum void it is influenced by scattering on the virtual electron-positron pairs. The relaxation time \(\tau\) can be calculated as:

\[
\tau^{-1} = (\tau_{e-p}^{-1} + \ldots + \tau_{Planck}^{-1}) \tag{23}
\]

where, for example, \(\tau_{e-p}\) denotes the scattering of the particle \(m_i\) on the electron-positron pair \((\tau_{e-p} \sim 10^{-17} \text{ s})\) and the shortest relaxation time \(\tau_{Planck}\) is the Planck time \((\tau_{Planck} \sim 10^{-43} \text{ s})\).

From equation (23) we conclude that \(\tau \approx \tau_{Planck}\) and equation (21) can be written as

\[
\frac{i\hbar}{\tau} \frac{\partial \Psi}{\partial \tau} = \nabla \Psi - \frac{\hbar^2}{2m_i} \nabla^2 \Psi - \tau_{Planck} \frac{\hbar}{\tau} \frac{\partial^2 \Psi}{\partial \tau^2}, \tag{24}
\]

where

\[
\tau_{Planck} = \left(\frac{\hbar G}{c^5}\right)^{\frac{1}{2}} = \frac{\hbar}{2M_p c^2}. \tag{25}
\]

In formula (25) \(M_p\) is the mass Planck. Considering Eq. (25), Eq. (24) can be written as

\[
\frac{i\hbar}{\tau} \frac{\partial \Psi}{\partial \tau} = -\frac{\hbar^2}{2m_i} \nabla^2 \Psi + V\Psi - \frac{\hbar}{\tau_{Planck}} \frac{\partial^2 \Psi}{\partial \tau^2}, \tag{26}
\]

The last two terms in Eq. (26) can be defined as the Bohmian pilot wave

\[
\frac{\hbar^2}{2M_p} \nabla^2 \Psi - \frac{\hbar^2}{2M_p c^2} \frac{\partial^2 \Psi}{\partial \tau^2} = 0, \tag{27}
\]
i.e.,

\[
\nabla^2 \Psi - \frac{\hbar}{c^2} \frac{\partial^2 \Psi}{\partial \tau^2} = 0. \tag{28}
\]

It is interesting to observe that pilot wave \(\Psi\) does not depend on the mass of the particle. With postulate (28) we obtain from equation (26)

\[
\frac{i\hbar}{\tau} \frac{\partial \Psi}{\partial \tau} = -\frac{\hbar^2}{2m_i} \nabla^2 \Psi + V\Psi - \frac{\hbar^2}{2M_p} \nabla^2 \Psi \tag{29}
\]
and simultaneously

\[
\frac{\hbar^2}{2M_p} \nabla^2 \Psi - \frac{\hbar^2}{2M_p c^2} \frac{\partial^2 \Psi}{\partial \tau^2} = 0. \tag{30}
\]

In the operator form Eq. (21) can be written as

\[
\hat{E} = \hat{\mathbf{p}}^2 + \frac{1}{2m_i} \hat{\mathbf{P}}^2 + \frac{\hbar^2}{2M_p} \frac{\partial^2 \Psi}{\partial \tau^2}, \tag{31}
\]

where \(\hat{\mathbf{E}}\) and \(\hat{\mathbf{p}}\) denote the operators for energy and momentum of the particle with mass \(m_i\). Equation (31) is the new dispersion relation for quantum particle with mass \(m_i\). From Eq. (21) one can conclude that Schrödinger quantum mechanics is valid for particles with mass \(m_i \ll M_p\). But pilot wave exists independent of the mass of the particles.

For particles with mass \(m_i \ll M_p\) Eq. (29) has the form

\[
\frac{i\hbar}{\tau} \frac{\partial \Psi}{\partial \tau} = -\frac{\hbar^2}{2m_i} \nabla^2 \Psi + V\Psi. \tag{32}
\]

In the case when \(m_i \approx M_p\), Eq. (29) can be written as
\[ i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2M_p} \nabla^2 \Psi + V\Psi, \quad (33) \]

but considering Eq. (30) one obtains
\[ i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2M_p} \frac{\partial^2 \Psi}{\partial t^2} + V\Psi \quad (34) \]

or
\[ \frac{\hbar^2}{2M_p} \frac{\partial^2 \Psi}{\partial t^2} + i\hbar \frac{\partial \Psi}{\partial t} - V\Psi = 0. \quad (35) \]

We look for the solution of Eq. (35) in the form
\[ \Psi(x,t) = e^{i\omega t} u(x). \quad (36) \]

After substitution formula (16) to Eq. (35) we obtain
\[ \frac{\hbar^2}{2M_p} \omega^2 + \omega \hbar + V(x) = 0 \quad (37) \]

with the solution
\[ \omega_1 = \frac{-M_p c^2 + M_p c^2 \sqrt{1 - \frac{2V}{M_p c^2}}}{\hbar}, \]
\[ \omega_2 = \frac{-M_p c^2 - M_p c^2 \sqrt{1 - \frac{2V}{M_p c^2}}}{\hbar} \]

for \( \frac{M_p c^2}{2} > V \) and
\[ \omega_1 = \frac{-M_p c^2 + iM_p c^2 \sqrt{\frac{2V}{M_p c^2} - 1}}{\hbar}, \]
\[ \omega_2 = \frac{-M_p c^2 - iM_p c^2 \sqrt{\frac{2V}{M_p c^2} - 1}}{\hbar} \]

for \( \frac{M_p c^2}{2} < V \).

Both formulae (38) and (39) describe the string oscillation, formula (27) damped oscillation and formula (28) over damped string oscillation.

#### 3. Gravity and Schrödinger Equation

Classically, when the inertial mass \( m_i \) and the gravitational mass \( m_g \) are equated the mass drops out of Newton’s equation of motion, implying that particles of different mass with the same initial condition follows the same trajectories. But in Schrödinger’s equation the masses do not cancel. For example in a uniform gravitational field (Kozłowski, 1997)
\[ i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m_i} \frac{\partial^2 \Psi}{\partial x^2} + m_g \partial^2 \Psi \quad (40) \]

implying mass dependent difference in motion.

In this paragraph we investigate the motion of particle with inertial mass \( m_i \) in the potential field \( V \). The potential \( V \) contains all the possible interactions including the gravity.

\[ i\hbar \frac{\partial \Psi}{\partial t} = V\Psi - \frac{\hbar^2}{2m_i} \frac{\partial^2 \Psi}{\partial x^2} - 2\hbar \frac{\partial^2 \Psi}{\partial t^2} \quad (41) \]

where the term
\[ 2\hbar \frac{\partial^2 \Psi}{\partial t^2}, \quad \tau = \frac{\hbar}{m_i c} \quad (42) \]

describes the memory of the particle with mass \( m_i \). Above equation for the wave function \( \Psi \) is the local equation with finite invariant speed, \( c \) which equals the light speed in the vacuum.

Let us look for the solution of the Eq. (40), \( V=0 \), in the form (for 1D)
\[ \Psi = \Psi(x-ct). \quad (43) \]

For \( \tau \neq 0 \), i.e. for finite Planck mass we obtain:
\[ \Psi(x-ct) = \exp\left(\frac{2\mu c}{\hbar}(x-ct)\right) \quad (44) \]

where the reduced \( \mu \) mass equals
\[ \mu = \frac{m_i M_p}{m_i + M_p} \quad (45) \]
For $m_i \ll M_p$, i.e. for all elementary particles one obtains
\begin{equation}
\mu = m_i \tag{46}
\end{equation}
and formula (44) describes the wave function for free Schrödinger particles
\begin{equation}
\Psi(x-ct) = \exp\left(\frac{2mi c}{\hbar}(x-ct)\right) \tag{47}
\end{equation}
For $m_i \gg M_p$, $\mu = M_p$
\begin{equation}
\Psi(x-ct) = \exp\left(\frac{2M_i c}{\hbar}(x-ct)\right) \tag{48}
\end{equation}
From formula (48) we conclude that $\Psi(x-ct)$ is independent of mass $m_i$. In the case $m_i < M_p$ from formulae (45) and (46) one obtains
\begin{equation}
\mu = m_i \left(1 - \frac{m_i}{M_p}\right) \tag{49}
\end{equation}
\begin{equation}
\Psi(x-ct) = \exp\left(\frac{2im_i c}{\hbar}(x-ct)\right) \tag{49}
\end{equation}
\begin{equation}
\exp\left(-i \frac{m_i}{M_p} \frac{2m_i c}{\hbar} x - \frac{2m_i c^2}{\hbar} t\right) \tag{49}
\end{equation}
In formula (49) we put
\begin{equation}
k = \frac{2m_i c}{\hbar} \tag{50}
\end{equation}
\begin{equation}\omega = \frac{2m_i c^2}{\hbar} \tag{50}\end{equation}
and obtain
\begin{equation}
\Psi(x-ct) = e^{ikx-\omega t} e^{-i \frac{m_i}{M_p} (kx-\omega t)} \tag{51}
\end{equation}
As can concluded from formula (51) the second term depends on the gravity
\begin{equation}
\exp\left[-i \frac{m_i}{M_p} (kx-\omega t)\right] = \tag{52}
\end{equation}
\begin{equation}
\exp\left[-i \left(\frac{m_i^2 G}{\hbar c}\right)^\frac{1}{2} (kx-\omega t)\right] \tag{52}
\end{equation}
where $G$ is the Newton gravity constant.
It is interesting to observe that the new constant, $\alpha_G$,
\begin{equation}
\alpha_G = \frac{m_i^2 G}{\hbar c} \tag{53}
\end{equation}
is the gravitational constant. For $m_i = m_N$ nucleon mass
\begin{equation}
\alpha_G = 5.9042 \cdot 10^{-39} \tag{54}
\end{equation}

4. Quantum $\rightarrow$ classical transition in the brain
As was shown in paragraph 2 the transition quantum $\rightarrow$ classical behavior occurs at mass $\approx 10^{-5}$ g, i.e. at the mass of the order of the human neuron mass $\approx$ Planck mass.

The Planck mass depends on the Planck constant $\hbar$, light velocity $c$ and gravity constant $G$,
\begin{equation}
M_p = \sqrt{\frac{\hbar c}{G}}. \tag{55}
\end{equation}
In the papers (Kozłowski, 1997; Kozłowski, 1999) the Klein – Gordon equation for Planck gas, i.e., gas of particles with mass was formulated and solved. As the one of the fundamental result – the time arrow creation by gravity was stated.

Considering the result of the present paper and the results of the papers (Kozłowski, 1997; Kozłowski, 1999) it can be argued that the gravitation plays the fundamental role in the brain operation:
1. The gravity, i.e., when $G \neq 0$, creates the arrow of time.
2. The gravity, through the Planck mass is responsible for the quantum $\rightarrow$ classical transition in the brain. The brain can be considered as the fluid which consists of the Planck particles, i.e. the Planck fluid with very high viscosity
3. An understanding of the basic underlying fluid mechanics in the normal brain and in hydrocephalus is needed to develop more effective medical treatments. In particular, the pulsatile flow rates, pressure gradients, and their interaction with each other need to be understood in order to design CSF (cerebral-spinal fluid) shunting systems which compensate correctly for the fluid dynamic abnormalities.
In standard model description (Linniger, 2005) the brain body flows in CSF. The system: brain and CSF forms the non-Newtonian fluid. Non-Newtonian fluid is the high viscosity fluid which can be described as the medium in between the fluid and solid state. It is interesting to observe that the transport phenomena in non-Newtonian fluids are described by hyperbolic transport equation, the same as the equations (8, 21).

4. The hyperbolic Schrödinger equation (21) is the local quantum equation i.e., the equation with the finite light velocity, c. The standard parabolic Schrödinger equation is non-local with infinite c and is in opposition to special relativity theory.

Conclusion
Cognitive neuroscience has made remarkable advances in recent years in understanding the neuronal basis of cognition and consciousness. But as is well, unsolved problems remain. Traditional quantum theory correctly predicts the results of a wide range of measurements. But as is well known it gives rise to a number of philosophical problems and paradoxes.

In this paper we developed the new local Schrödinger equation (MSE) which avoids the fundamental problem of the standard parabolic equation the non-locality due to infinite velocity of light c. The MSE opens the possibility of the inclusion of the gravity to the neuroscience.

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