SELECTION OF EFFICIENT EQUITY MUTUAL FUNDS PORTFOLIO IN INDIAN MARKET USING OPTIMIZATION TECHNIQUES.

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Abstract

This study aims to determine the efficient Indian equity mutual funds portfolio by allocating the weights dynamically ensuring minimum risk. All the equity mutual funds with inception before January 2015 are considered and their monthly returns are calculated. The funds with positive average return and negative skewness are taken into consideration. Since the performance of a mutual fund is compared with respect to the market, the portfolio is, then, constructed by taking the funds with low standard deviation and high beta value using the investor's perception map. BSE 100 is taken as the market benchmark and their monthly returns for the same period are calculated. Appropriate weightage has been allocated among the funds belonging to the portfolio using the Generalized Reduced Gradient method ensuring minimum risk. Based on market return parameters the efficiency scores of the selected funds are calculated using Data Envelopment Analysis to verify whether the funds constituting the portfolio are efficient. This study will help the investors to choose mutual funds wisely as well as instruct them on how to distribute a proper investment weighting, which will aid them in making future investment decisions.

Keywords: Equity Mutual Funds,Portfolio,Return, Skewness, Generalized Reduced GradientMethod, Data Envelopment Analysis.

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1. Introduction

Indian capital market provides numerous investment avenues to the investors, to assist them to take an investment position in various industries and to confirm the profitable outcome on return on investment. Among different financial products, a mutual fund ensures minimum risk with maximum return to the investors. The Indian mutual fund industry provides an abundance of schemes and serves all types of investor needs. There are currently as many as 44 registered fund houses in India that together offer more than 2,500 mutual fund schemes(Association of Mutual Funds in India (AMFI), (2020)). According to AMFI, the Indian mutual fund industry has registered a more than four-and-a-half-fold increase in a period of 10 years from 2009 - 2019. The returns on broader schemes of these funds from 2013-14 are higher than the other financial instruments like post office monthly income schemes, National Savings Certificate VIII issue, etc. (CRISIL- AMFI Mutual Fund Performance Indices, (2018)) which have shown a significant downfall in the interest rates during these periods (RBI (2018)).Because of its higher return as compared to the other conventional financial instruments, the mutual fund has been a profitable investment avenue for investors and after demonetization with banks slashing the interest rates, made more people turn to mutual funds.Equity mutual funds received massive net inflows of Rs 1.23 lakh crore in the first year itself after demonetization (CMIE (2017)). With this rapid increase of the mutual fund demand, close monitoring of mutual funds has become very essential, and choosing lucrative mutual funds for investment is a very important issue.

The selection of an efficient Portfolio is all about utilizing the entire capital optimally in terms of allocation of the funds belonging to the portfolio and balancing the same taking return – risk trade-off into account. It helps to reduce risks, maximizes efficiency, and enables repeatable success.Most of the previous studies of optimal portfolio allocation rely on the traditional mean-variance optimization according to the short-term returns (MacKinnon et al.(2009)). Despite there being many famous portfolio optimization models and techniques, the Modern Portfolio Theory (Markowitz(1952))tends to remain the most used model due to its simplicity and also because it analyses covariance, unlike other models. The mean-variance is one of the most important theories in finance studies where the Markowitz Efficient Frontier refers to the set of portfolios in which maximum expected returns are reached at a given level of risk (Lee et al., (2010)). The Modern Portfolio theory insisted on not only looking at the expected risk or the return of stock on an individual basis rather an investor should invest in multiple stocks to get benefit from diversification – thus leading to a reduction in the riskof the portfolio. Therefore, the objective of this study is to determine the efficient Indian equity mutual funds portfolio by allocating the weights dynamically to the funds ensuring minimum risk.

The rest of the paper is structured as follows. The next section revisits the existing literature followed by the methodological framework used for the study. Section IVgives the results and discusses the findings. Finally, section Vconcludes thestudy highlighting some important further scope of research.

2. Literature Review

There is a vast amount of mutual fund literature both at the Indian and global levels regarding the performance of various funds and the selection of funds for creating a portfolio using various performance evaluation measures. Redman et al. (2000) examined the risk-adjusted returns for five portfolios of international mutual funds using Sharpe's Index, Treynor's Index, and Jensen's Alpha. The study was conducted for three time periods and the benchmarks for comparison were the Vanguard Index 500 mutual fund and a portfolio of funds that invest solely in U. S. issued stocks. The performance of actively managed U.S. mutual funds from 1984 to 1999 was studied by Kasperczyk et al. (2005). Their results suggested that funds with concentrated portfolios perform better than those with diversified portfolios and the finding is robust to different risk-adjusted performance measures. Baballos et al. (2015) examined the performance of US no-load equity mutual funds using stochastic frontier analysis and they inferred that the funds exhibitdifferent levels of efficiency over time depending on size and investment style. The returns of large and small funds in Australia were studied by Gallagher and Martin (2005) and they found that there is no significant difference in the returns. Pastor and Stambaugh (2002) suggested a framework in which beliefs about managerial skills and pricing models are merged to select portfolios of equity mutual funds with maximum Sharpe Ratio. Thanou (2008) used the Treynor and Sharp indexes for risk-adjusted performance evaluation of each of 17 Greek Equity Mutual funds between 1997 and 2005 and inferred that most of the funds were in agreement with closely the market, accomplish comprehensive adequate diversification and some consistently outplayed the market. A multi-objective portfolio optimization had been proposed by Alimi et al. (2012) to provide asset allocation for mutual funds. The suggested model clusters mutual funds based on the Treynor index and Sharpe index and it uses fuzzy variables for return rate and semi-variance. Pareto optimal solutions are also obtained taking distinct weights for objective functions. Chen and Huang (2009) investigated the performance of Taiwanese listed equity mutual funds to determine the portfolio of equity mutual funds to gain optimal return rates. The funds were divided into four clusters, namely, inferior performance, stable, aggressive, and good performance funds based on rates of return, turnover rate, standard deviation, and Treynor index and their results suggested that aggressive and good performance funds overshadow the other two. The fuzzy optimization model was then suggested to find the optimal investment proportion of these two clusters. Similar work had been done by Kilicman and Sivalingam (2010) on the equity mutual funds of selected three Malaysian banks and their findings suggested that the performance of funds belonging to stable and good performing dominates the other clusters. Azis et al. (2017) had shown that mutual fund investment portfolios can be optimized through good corporate governance and financial banking performance.

In the Indian context also, we observed several studies on creating an optimum portfolio of funds. Tripathi (2004) evaluated the performance of 31 tax saving mutual fund schemes in India over the period 1994-1995 to 2001-2002 using the risk-adjusted measures, namely, Sharpe, Treynor, and Jensen measure, and her result suggested that the fund managers have not been successful in receiving returns above the market or in securing an efficient portfolio diversification. A similar study was conducted by Arora (2015) on a sample of 100 Indian mutual funds selected during 2000-2008, covering different types of funds. Prajapati and Patel (2012) made a relative study on the performance assessment of some selected diversified equity mutual fund schemes of Indian companies. The result suggested that most of the funds have given positive returns from 2007 to 2011 and among them, HDFC and Reliance mutual funds have performed well as compared to the Sensex return. The equity funds of the five fund houses, namely, ICICI Prudential, Canara Robeco, Franklin Templeton, Reliance, and L&T was analyzed by Rajani (2016) and he observed that L&T and Reliance give the highest returns for a low-risk and risk-taking investor respectively.Ray and Majumdar (2017) presented fuzzy portfolio selection models that maximize return and skewness and on the other side minimizes variance and cross-entropy. Rather et al. (2017) surveyed important articles on prediction and portfolio selection of stocks and from the extracted information from each of the articles, they concluded that prediction-based portfolio models perform well in portfolio selection of stocks. An ensemble procedure based on a two-stage methodology for portfolio optimization of the Indian open-ended large-cap funds (direct plan) was adopted by Biswas et al. (2019). The efficiencies of the funds are analyzed using Data Envelopment Analysis for primary selection of the funds and then the selected funds have been ranked based on risk and return parameters for investment portfolio formulation using Multi-Attribute Border Approximation Area Comparisons (MABAC) approach. Maji et al. (2021) developed a robust mutual fund portfolio over time based on regression analysis to gain profit over time and avoid risks. The portfolio is constructed by taking the closing stock prices of 20 different companies from BSE and the performance is compared with other CRISIL ranked mutual funds and benchmark indexes. The results suggested that the portfolio performs as well as those top-performing mutual funds

Moreover, applied operations research techniques and optimization techniques have been used by several researchers in finding an optimum portfolio of funds. Sharma and Sharma (2004) applied the lexicographic goal programming model for selecting an optimum mutual fund portfolio for an investor. The method of goal programming was also used by Jana and Panda (2017) to optimize the constructed portfolio. A two-stage methodology was adopted by Pendaraki et al. (2005) on the data collected from Greek mutual funds over the period 1999-2001. In the first stage, UTilités Additives DIScriminantes (UTADIS) multi-criteria decision aid method wasused to build mutual fund's performance models for the selection of mutual funds portfolio. In the second stage, a goal programming model was applied to determine the share of the selected funds in the final portfolios. Alptekin (2009) used Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS)to compare the performance of Turkish Type A mutual funds and Pension Stock funds and the findings suggested that Pension Stock Mutual Funds have superior performance than the Type A Mutual funds. The Preference Ranking Organization Method for Enrichment Evaluations to develop outranking model was used by Pendaraki and Zopounidis (2003) for the performance evaluation of Greek mutual funds. Lin et al. (2007) evaluated the performance of 82 Taiwanese mutual funds by investigating Treynor Ratio, Sharpe ratio, Information ratio, and Jensen's alpha separately and then combining all the indices in making a final ranking of the funds using TOPSIS. Alrabadi (2016) applied the portfolio optimization concept of Markowitz (1952) and the Generalized Reduced Gradient (GRG) algorithm to a portfolio comprising of the 30 prime stocks from the three different sectors in Amman Stock Exchange during 2009-2013 and concluded that the selected portfolios attain a monthly return of 5 percent while keeping risk at a minimum. The samemethodwasapplied by Li and Chan (2018) to investigate the optimum portfolio of Real Estate Investment Trust (REIT) price return of Asia –Pacific, North America, and a global portfolio with Asia –Pacific and North America countries. Empirical results showed that the performance of the global optimal portfolio is better than using Asia–Pacific countries or North American countries alone. They also suggested investing the largest portfolio proportion in Taiwan REITs, followed by Malaysia REITs, Korea REITs, Thailand REITs, and Canada REITs. Gupta et al. (2019) also applied the Generalized Reduced Gradient method in finding the individual weightage of stocks for the construction of a portfolio basket.

From the existing literature, we see that a plethora of research has been conducted on the selection of profitable mutual funds to create a portfolio. Apart from the usual risk-adjusted measures for performance evaluation, we have found the use of various multi-criteria decision-making techniques, linear and non-linear optimization techniques in finding an optimum portfolio of funds. We have observed that the existing literature regarding portfolio optimization of mutual funds in the Indian market suffers from the fact that they are considering small sample sizes or funds from some selected houses or making portfolios based on the performance of the funds only. However, the selection of an efficient portfolio by allocating weights to the funds constituting the portfolio is neither well documented nor explained in the finance literature, both at the Indian and global markets. Markowitz (1952) explained that investment is not about picking stocks, but about selecting the proper combination of stocks among which to allocate the risk associated. Thus, in this study, an attempt has been made to select the efficient Indian Equity Mutual Funds portfolio by appropriate weight allotment to individual funds constituting the portfolio ensuring minimum risk using suitable optimization technique. This would be a first attempt to use optimization techniques to select an efficient portfolio by suitable weight assignment to individual funds constituting the portfolio ensuring minimum risk.

3. Materials and Methods

Equity mutual funds invest more than 60% of their assets in equity shares of numerous companies in suitable proportion and they are highly preferred by the investors (Gandhi and Joshi (2018)). They can be categorized mainly into three types based on the market capitalization of the companies they invest in i) Large Cap; ii) Mid Cap; iii) Small Cap.LargeCap funds are essentially less risky& volatile than their MidCap counterparts, and MidCap funds are in turn less risky& volatile than SmallCap funds. However, Mid and SmallCap funds tend to offer more growth potential than LargeCap funds. (GROWW)

3.1 Data

The sample for this study consists of Large Cap, Mid Cap, and Small Cap mutual fundswith at least 5 years of operation in the Indian mutual fund market. The period of the study is January 2015 to December 2019. Therefore, we have included only those funds which are operational on or before January 2015. There are altogether 222 such funds and their monthly Net Asset Values (NAV) fromJanuary 2015 to December 2019 are collected(AMFI). The monthly returns are then calculated based on changes in their NAV values over time given as

R_t= (NAV_t- NAV_{t-1}) / NAV_{t-1}, t = 2, 3, …, 60 (Baliyan and Rathi, 2019)

where R_t: Return for month t and NAV_t and NAV_{t-1}: Net asset value formonth t and t-1 respectively.(1)

3.2 Selection of Funds

The funds which have negative average returns areeliminated. The average monthly return of each fund is calculated as$\bar{x}$ = $\sum_1^n x_i$ / n; x_i = return of the i^{th} month of the fund. Here n = 60. Then we consider those funds which have negative skewness. Skewness is measured by Bowley's formula:

Sk = [(Q_3 – Q_2)– (Q_2 – Q_1)] / [(Q_3 – Q_2) + (Q_2 – Q_1)],

where Q_i: ith quartile of the distribution, i = 1,2,3. A negatively skewed distribution is a type of distribution in which more values are concentrated on the right side (tail) of the distribution graph while the left tail of the distribution graph is longer.The negative skewness of the distribution indicates that an investor may expect few large losses. Many trading strategies employed by traders are based on negatively skewed distributions as they provide stable profits and higher returns with time (Corporate Finance Institute).

We then calculate the Standard deviation of the monthly returnsand Beta of each fund.

Standard deviation (SD) is a measure of an investment's total risk. It includes both systematic and unique risks. The deviation of the returns from their average return is expressed by standard deviation. It is defined asSD = $[\sum_1^n (x_i - \bar{x})^2 / 59\ ]^{1/2}$. (2)

Beta is a measure of an investment's systematic risk compared to the market. It is calculated using the formula:

beta = Cov (x,y) / Var(y),

whereCov (x, y) = $\sum_{i,j=1}^{n} (x_i - \bar{x})(y_i - \bar{y})$ /59 and Var(y) = $\sum_{i=1}^{n} (y_i - \bar{y})^2$ / 59. (3)

Here x_i = return of the i^{th} month of the fund, y_i =market return of the i^{th} month and $\bar{x}$ and $\bar{y}$ denote respectively their average returns.

In our study, Bombay Stock Exchange (BSE) 100 is considered as the market benchmark. BSE is one of the leading stock exchanges of the Indian market and the oldest stock exchange marketplace not just for India but Asia as well, which offers high-speed trading to its customers.

Next, we select those funds whose SD is less than the combined SD and beta is greater than the average beta. This is done by drawing Investor's Perception Map taking SD in X-axis and beta in Y-axis. Investor's Perception Map displays the position of betaand SDof a fund with respect to average beta and combinedSD. Thus, the funds which fall in the second quadrant of the map are considered.

The combined SDis calculated by the formula:

combined SD = $(\sum n_i . s_i^2 + \sum n_i . (\bar{x}_i - \bar{x})^2 / \sum n_i)^{1/2}$(Goon, Gupta and Dasgupta, 2008)

where s_i^2 = variance of monthly returns of i^{th} fund, $\bar{x}_i$ = average monthly returns ofi^{th} fund, $\bar{x}$ = combined average and n_i = number of monthly returns for each fund. (4)

The average beta is given by average beta = sum of the beta values / n, where n = number of fundsremaining at this stage. (5)

In the next step, we have allocated weights to the selected funds to minimize the portfolio risk.

According to the Modern Portfolio Theory, the portfolio risk is given by

$\sigma = \{\sum \text{Var}(w_i . R_i)\}^{1/2} = \{\sum w_i^2 . \sigma_i^2 + \sum\sum w_i . w_j . \sigma_{ij}\}^{1/2}$

where R_i and R_j are the monthly returns of the ith and jth fund, w_i and w_j are the proportion of the portfolio invested in the ith and jth fund, σ_i^2 = Var(R_i) and σ_{ij} = Cov(R_i, R_j) = $\sum_{i,j} (R_i - \bar{R}_i)$. $(R_j - \bar{R}_j)$ / n-1. (Markowitz, 1952)

The portfolio risk is non-linear and hence to find the optimal value of $w_i^{'}$s, we have used a non-linear optimization technique - Generalized Reduced Gradient method.

3.3 Generalized Reduced Gradient Method

Lasdon, Fox, Ratner (1974) proposed the Generalized Reduced Gradient (GRG) method, which is one of the most popular methods to solve non-linear optimization problems (Chapra and Canale**,** 2011). GRG is the most widely used and efficient method for portfolio optimization. Many researchers have used this method to form optimal portfolios viz. Alrabadi (2016), Li and Chan (2018), Gupta et al. (2019), etc. The main idea of this method is to solve the non-linear problem contend with effective inequalities. It only requires the objective function to be differentiable. The variables are segregated into a set of basic (dependent) variables and non-basic (independent) variables and the reduced gradient is computed to find the minimum in the search direction. This process is repeated until convergence is obtained.

Lasdon, Fox, Ratner (1974) discusses the basic principles of GRG and constructs a specific GRG algorithm as follows.

Minimize f(X) (6)
Subject to the constraints $g_i(X) = 0$, i= 1,2,..,m (7) and
$l_j \le X_j \le u_j$, j= 1,2,…,n(8)

where X is an n-dimensional vector and l_j and u_j are given lower and upper bounds, assuming m < n to avoid infeasibility of solutions or unique solutions. (6) – (8) is general, since inequality constraints are transformed to equalities (7) by adding slack variables. The basic idea of GRG is to use the equalities (7) to express m variables, called the basic variables, in terms of the remaining n-m non-basic variables. Let $\bar{x}$ be a feasible point and y be the vector of basic variables and z the non basic at $\bar{x}$, so that X is partitioned as X= (y,z), $\bar{X}$ = ($\bar{y}$,$\bar{z}$) and g(y,z) = 0, where g = (g_1, g_2, …, g_m). Here it is assumed that both the objective function and constraints are differentiable. The transformed objective function is then given as F(z) = f (y(z),z) and accordingly the non-linear problem is transformed at least for z close to $\bar{z}$, to a reduced problem with lower and upper bounds given by:

Minimize F(z) (9)
Subject to $l_{NB} \le z \le u_{NB}$ (10)

where l_{NB} and u_{NB} are the vectors of bounds for z. GRG solves the original problem (6) – (8) by solving a sequence of problems of the form (9) – (10) which may be solved by the simple modifications of unconstrained minimization.
Here, our non-linear optimization problem is stated below
Min $\sigma = \{\sum w_i^2 . \sigma_i^2 + \sum\sum w_i.w_j.\sigma_{ij}\}^{1/2}$
subject to
$\sum w_i = 1$ and $w_i \ge 0$ for all i; where w_i and w_j are the proportion of the portfolio invested in the ith and jth fund, σ_i^2 = Var(R_i) and σ_{ij} = Cov(R_i, R_j) = $\sum_{i,j} (R_i - \bar{R}_i).(R_j - \bar{R}_j)$ / n-1.

Finally, the efficiency scores of thefunds are calculated based on market return parameters usinga non-parametric mathematical programming model Data Envelopment Analysis (DEA) to verify whether the funds constituting the portfolio are efficient.

3.4 Data Envelopment Analysis

Charnes et al. (1978) developed DEA for measuring the relative efficiency of a group of entities or decision-making units (DMUs) where the form of the production is not known.DEA is based on linear programming which makes it superior to any other productivity management tools (Cooper et al. (2011)). The DEA model can be developed toeither minimize inputs or to maximize outputs.DEA has adequate pertinency in finance literature in various fields, which includes the evaluation of investment funds (Morey and Morey, (1999); Basso and Funari, (2001); Haslem and Scheraga, (2006); Novickyte and Drozdz, (2018)), construction of portfolios (Powers and McMullen, (2000); Bao et al.,(2008); Lopes et al.,(2008)) although its initial applications had been primarily to public organizations (Avkiran, (2001),Li et al. (2005)). However, apart from finance literature, DEA is widely used in many other fields viz. Ramanathan (2001), Sarkis and Seol (2010), Louis and Joel (2010), Venkatesh and Kushwaha (2017), Daultani et al. (2021), etc.DEA itself doesn't furnish any guidance in choosing the input and output variables and this selection is left to the researchers.In this study, we take SD as the input variable and beta as the output variable.

4. Results and Discussions

The monthly returns of 222 funds are calculated using equation (1).After consideringthe funds with positive average return and negative skewness of returns,we have 68 funds (26 Large Cap, 20 Mid Cap, and 22 Small Cap). The average monthly returns, the standard deviation of monthly returns, and the beta values of these 68 funds are given in the annexure.

The combined SD and the average beta are respectively given by 0.045 and 0.622, using equations (2) and (3). The investor's perception map is prepared taking SD in X-axis and beta in Y-axis with its origin at average beta (0.622) and combined SD (0.045).

Figure 1. Perception Map

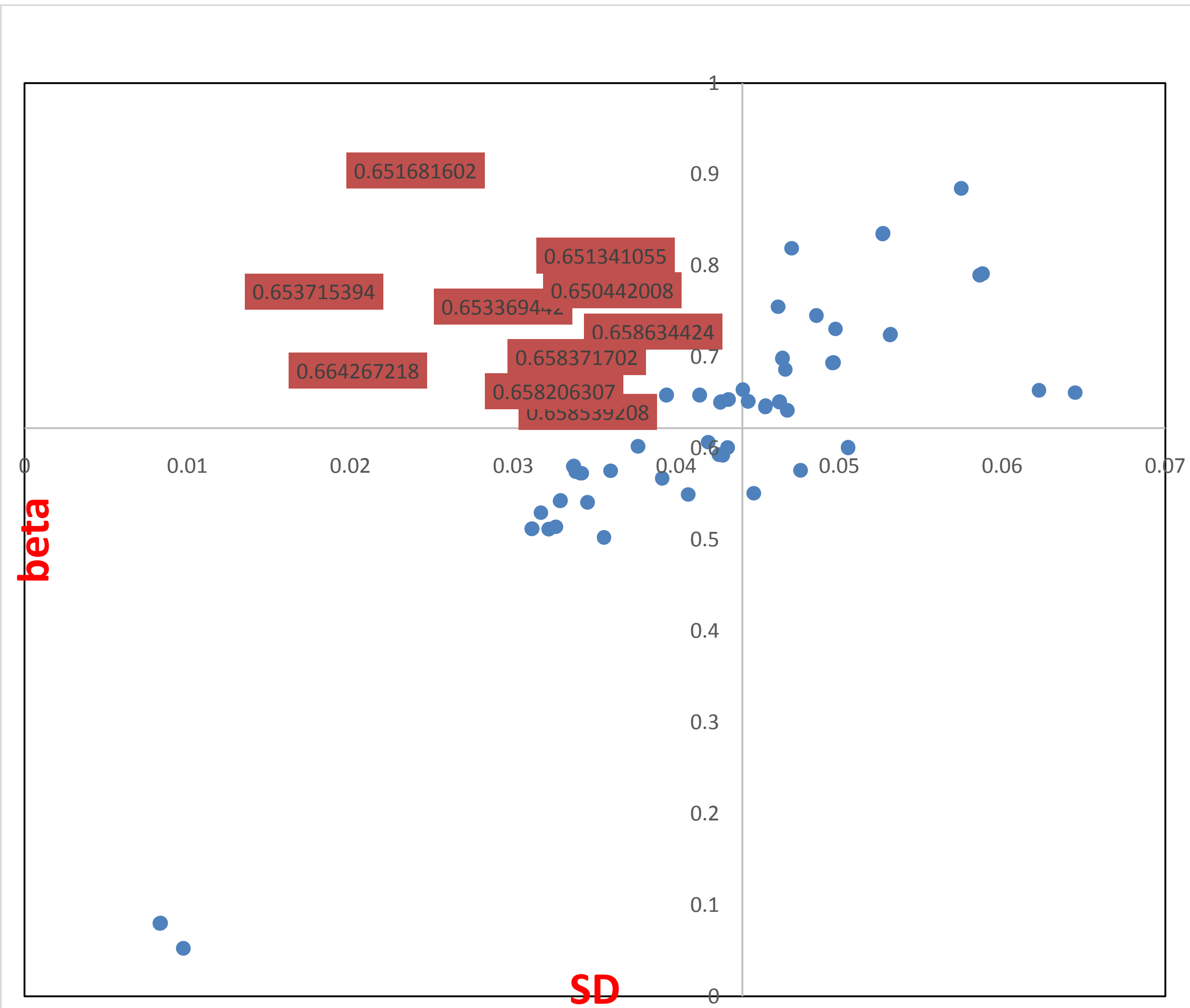

Table 1. List of funds belonging to the second quadrant along with their SDand beta

Category	Funds	SD	Beta
Large Cap Fund	ICICI Pru Nifty Next 50 Index - D (G)	0.041443	0.658539
Large Cap Fund	ICICI Prudential Nifty Next 50 Index (G)	0.041426	0.658206
Large Cap Fund	HDFC Top 100 Fund - Direct (G)	0.039406	0.658634
Large Cap Fund	HDFC Top 100 Fund (G)	0.039391	0.658372
Mid Cap Fund	L&T Midcap Fund -Direct (G)	0.043211	0.653715
Mid Cap Fund	L&T Midcap Fund (G)	0.043177	0.653369
Mid Cap Fund	ABSL Midcap Fund -Direct (G)	0.042749	0.651341
Mid Cap Fund	Taurus Discovery (Midcap) - Direct (G)	0.044086	0.664267
Mid Cap Fund	ABSL Midcap Fund (G)	0.042706	0.650442
Mid Cap Fund	UTI Mid Cap (G)	0.044421	0.651682

Source: authors calculation

The beta values of all these funds are less than 1 implying that all the funds are less volatile than the market. We also note that none of the Small Cap funds falls in the second quadrant and thus are not considered in the portfolio.

The variance-covariance matrix of the monthly returns of the selected funds is then computed to minimize the portfolio risk.

Table 2. Variance Covariance Matrix

0.001688									
0.001688	0.001687								
0.001469	0.001468	0.001527							
0.001469	0.001468	0.001526	0.001525						
0.001638	0.001638	0.00148	0.00148	0.001836					
0.001637	0.001636	0.001479	0.001479	0.001834	0.001833				
0.001631	0.001631	0.001455	0.001455	0.001773	0.001772	0.001797			
0.001709	0.001709	0.001532	0.001531	0.001808	0.001807	0.001784	0.001911		

0.001629	0.001629	0.001454	0.001454	0.001771	0.001769	0.001795	0.001781	0.001793	
0.001646	0.001645	0.001468	0.001468	0.001809	0.001808	0.001768	0.001813	0.001766	0.00194

Source: author’s calculation

With the values of the variance’s and covariance’s, we then proceed to find the weights of the individual funds using GRG.

Table 3: Weights of the funds using GRG

Funds	**Weights**
ICICI Pru Nifty Next 50 Index - D (G)	0
ICICI Prudential Nifty Next 50 Index (G)	12%
HDFC Top 100 Fund - Direct (G)	36%
HDFC Top 100 Fund (G)	41%
DSP Mid Cap Fund -Direct (G)	0
L&T Mid Cap Fund (G)	0
ABSL Mid Cap Fund -Direct (G)	0
Taurus Discovery (Mid Cap) - Direct (G)	0
ABSL Mid Cap Fund (G)	11%
UTI Mid Cap Fund (G)	0

Source: author’s calculation

We find that three large cap funds, namely HDFC Top 100 Fund(G), HDFC Top 100 Fund – Direct (G),ICICI Prudential Nifty Next 50 Index (G), and one mid cap fund namely, ABSL Midcap Fund (G) constitutes the portfolio. It is seen that both HDFC Top 100 Fund(G) and HDFC Top 100 Fund – D (G) can be considered as a very good investment option for risk-averse investors. The results obtained, thus agrees with the market theory that Large Cap funds are generally less risky than Mid Cap and Small Cap funds.

Using the weights and variance’s and covariance’s,the optimum risk of the portfolio is found to be 0.15%, which is fairly low.

The correlation matrix of the monthly returns of these four funds constituting the portfolio is constructed to find the association among them.

Table 4. Correlation Matrix

	ICICI Pru Nifty Next 50 Index (G)	HDFC Top 100 Fund - D (G)	HDFC Top 100 Fund (G)	ABSL Mid Cap Fund (G)
ICICI Prudential Nifty Next 50 Index (G)	1	0.914968	0.915082	0.909439
HDFC Top 100 Fund - D (G)		1	0.999998	0.853142
HDFC Top 100 Fund (G)			1	0.85364
ABSL Mid Cap Fund (G)				1

Source: author’s calculation

The value of the determinant is .0000071 ≈ 0.
Thus, it is evident thatthe portfolio is diversified, and thus unsystematic risk is eliminated. (Markowitz, (1952); Gupta et al., (2019))
Finally, we measure the efficiency of these selected10 funds based on market return parametersusing DEA. The DEAconceptneedsthenumberofDMUtobeequaltothemaximumnumberof multiplicationsofinput andoutputorthreetimes thesummationofinputsand outputs (RaabandLichty, (2002)).As we have oneinput and one output variable, hence the minimum number of DMU's (i.e funds) on which DEA is applied, should be max {1x1, 3(1+1)} = 6. For single input-output, the efficiency and relative efficiency of each fund are respectively defined as efficiency = output/input and relative efficiency = efficiency/maximum efficiency.
The efficiency, relative efficiencyof each of these funds and their ranks are given below

Table 5: Relative Efficiency scores of the funds and their ranks

Funds	SD (input)	Beta (output)	Efficiency	Relative Efficiency	Ranking
ICICI Pru Nifty Next 50 Index - D (G)	0.04144	0.65854	15.89023	0.950711	3
ICICI Pru Nifty Next 50 Index (G)	0.04143	0.65821	15.88872	0.95062	4
HDFC Top 100 Fund - Direct (G)	0.03941	0.65863	16.71405	1	1
HDFC Top 100 Fund (G)	0.03939	0.65837	16.71377	0.999983	2
DSP Mid Cap Fund -Direct (G)	0.04321	0.65372	15.12844	0.905133	8
L&T Mid Cap Fund (G)	0.04318	0.65337	15.13234	0.905366	7
ABSL Mid Cap Fund - Direct (G)	0.04275	0.65134	15.2364	0.911593	5
Taurus Discovery (Mid Cap) – D (G)	0.04409	0.66427	15.06753	0.901489	9
ABSL Mid Cap Fund (G)	0.04271	0.65044	15.23069	0.911251	6
UTI Mid Cap (G)	0.04442	0.65168	14.67058	0.87774	10

Source: author's calculation.

We see that HDFC Top 100 Fund - Direct (G) is the efficient fund. Also, from the ranking of the funds, it is evident that Large Cap funds are more efficient than their Mid Cap counterpart.

Conclusion

Mutual funds have become the most popular financial investment avenues for the diversity of investment since they can disperse investment risks to the smallest degreeThe key to gain good profit from mutual fund investment is to understand where and in what proportion to invest in. In this study, an efficient portfolio of Indian Equity Mutual funds has been determined through dynamic allocation of the weights to the funds to minimize the portfolio risk.

All the Equity funds consisting of Large Cap, Mid Cap, and Small Cap funds with at least 5 years of operation in the Indian Mutual Fund market have been considered.10 funds have been selected applying two-stage criteria'si.e., i) funds having positive average return and negative skewness of return andii) funds that fall in the second quadrant of the Investor's Perception Map.

Then, the portfolio is constructed by allocating weights to these selected funds ensuring minimum risk using a nonlinear optimization techniqueGRGmethod. Three Large Cap funds, HDFC Top 100 Fund (G), HDFC Top 100 Fund Direct (G), ICICI Prudential Nifty Next 50 Index (G), and one Mid Cap fund,ABSL Mid Cap Fund (G) constitute the portfolio with respective weightage 41%, 36%, 13%, and 11%. The risk of the portfolio using their weights and their variances –covariances is 0.15%, which is fairly low. Also, the funds constituting the portfolio are highly correlated which makes the portfolio diversified.

Finally, the efficiency of these funds is measured using a nonparametric mathematical programming model DEA. HDFC Top 100 Fund - Direct (G) is the efficient fundand thus ranked 1 according to DEA. It is to be noted that the three large Cap funds, namely, HDFC Top 100 Fund (G), HDFC Top 100 Fund – Direct (G), and ICICI Prudential Nifty Next 50 Index (G) which constitutes the portfolio with 41%, 36% and 13% respectively (as obtained from GRG method), has ranked very high according to DEA. The only Mid Cap fund, namely, ABSL Mid Cap fund (G) which constitutes the portfolio with 11% weightage, though ranked 6 in DEA, but has relative efficiency value of more than 91%.

Thus, the funds constituting the portfolio can well be considered as efficient funds. The results of this study will provide a broader perspective for the selection of the portfolio. This study will not only allow the investors to make a prudent selection of mutual funds but also guides them about the distribution of appropriate investment weightage, which will help them in formulating future investment decisions.Moreover, our results conform to the market-based rating of the funds as HDFC's top 100 funds, which have got the highest weightage in GRG and highest ranking in DEA, is one of the leading Indian equity mutual funds at present for an investor with a low-risk appetite. However, future research is required regarding forecasting the performance of these efficient funds with time by building appropriate statistical models. Also,asan investor generally does not want to restrict his / her investment in one type of funds only, hence it is important to construct a portfolio considering all types of funds viz. Equity, Debt, and Hybrid.

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Annexure

Table 6: Average monthly returns, SD of monthly returns, and Beta value of 68 funds

Category	Funds	Average	SD	Beta
Large Cap Funds	SBI Blue Chip Fund - Direct (G)	0.014868	0.041443	0.658539
Large Cap Funds	Reliance Large Cap Fund - RP (G)	0.014277	0.037656	0.60235
Large Cap Funds	ABSL Frontline Eqty-Direct (D)	0.013313	0.033714	0.580891
Large Cap Funds	ICICI Pru Nifty Next 50 Index - D (G)	0.013728	0.032179	0.571592
Large Cap Funds	ICICI Pru Nifty Next 50 Index (G)	0.014506	0.041426	0.658206
Large Cap Funds	ICICI Pru Bluechip Fund (G)	0.012301	0.031688	0.529681
Large Cap Funds	Kotak Bluechip Fund - D (G)	0.012632	0.033825	0.574684
Large Cap Funds	JM Core 11 Fund (G)	0.0153	0.047082	0.8192
Large Cap Funds	JM Core 11 Fund -Direct (G)	0.0153	0.047082	0.8192
Large Cap Funds	ABSL Frontline Equity (G)	0.012493	0.033685	0.580259
Large Cap Funds	UTI Master share Master Share - Direct (G)	0.012131	0.032874	0.542438
Large Cap Funds	CR Bluechip Equity Fund - D (G)	0.01214	0.034107	0.572809

Large Cap Funds	HDFC Top 100 Fund - D (G)	0.01308	0.039406	0.658634
Large Cap Funds	Franklin (I) Bluechip - Direct (G)	0.01141	0.031154	0.512189
Large Cap Funds	UTI Master share Master Share (G)	0.01162	0.0329	0.542906
Large Cap Funds	Essel Large Cap Equity - D (G)	0.011409	0.032604	0.514123
Large Cap Funds	L&T India Large Cap - Direct (G)	0.011674	0.034198	0.572865
Large Cap Funds	HDFC Top 100 Fund (G)	0.012496	0.039391	0.658372
Large Cap Funds	Franklin India Bluechip (G)	0.010688	0.031136	0.511939
Large Cap Funds	Taurus Largecap Equity Fund (D)	0.013134	0.048601	0.745632
Large Cap Funds	Edelweiss Large Cap - Direct (D)	0.008152	0.03597	0.575496
Large Cap Funds	SBI Blue Chip Fund (D)	0.007536	0.034549	0.541003
Large Cap Funds	HDFC Top 100 Fund - D (D)	0.005254	0.042615	0.593174
Large Cap Funds	Franklin (I) Bluechip - Direct (D)	0.004749	0.035567	0.502461
Large Cap Funds	HDFC Top 100 Fund (D)	0.00449	0.042855	0.592424
Large Cap Funds	ICICI Pru Bluechip Fund (D)	0.004542	0.039138	0.56721
Mid Cap Funds	L&T Midcap Fund -Direct (G)	0.020221	0.043211	0.653715
Mid Cap Funds	L&T Midcap Fund (G)	0.0195	0.043177	0.653369
Mid Cap Funds	ABSL Midcap Fund -Direct (G)	0.016938	0.042749	0.651341
Mid Cap Funds	UTI Mid Cap - Direct (G)	0.017347	0.044407	0.651568
Mid Cap Funds	DSP Mid Cap - Direct (G)	0.017816	0.046341	0.651359
Mid Cap Funds	DSP Mid Cap - Direct (D)	0.017815	0.046341	0.651363
Mid Cap Funds	Tata Mid Cap Growth - Direct (G)	0.017712	0.046525	0.69897

Mid Cap Funds	Taurus Discovery (Midcap) - D (G)	0.016973	0.044086	0.664267
Mid Cap Funds	DSP Mid Cap - Regular (G)	0.017178	0.046321	0.650938
Mid Cap Funds	ABSL Midcap Fund (G)	0.016202	0.042706	0.650442
Mid Cap Funds	UTI Mid Cap (G)	0.016618	0.044421	0.651682
Mid Cap Funds	Tata Mid Cap Growth Fund (G)	0.017043	0.046506	0.698486
Mid Cap Funds	Tata Mid Cap Growth - Direct (D)	0.016465	0.046689	0.686439
Mid Cap Funds	Taurus Discovery (Midcap) - D (D)	0.014981	0.045466	0.645417
Mid Cap Funds	Taurus Discovery (Midcap) (D)	0.014634	0.045483	0.64676
Mid Cap Funds	L&T Midcap Fund -Direct (D)	0.013459	0.046244	0.755207
Mid Cap Funds	Tata Mid Cap Growth Fund (D)	0.013134	0.048601	0.745632
Mid Cap Funds	Invesco India Midcap - D (D)	0.011181	0.04475	0.55091
Mid Cap Funds	UTI Mid Cap (I)	0.011564	0.049773	0.730908
Mid Cap Funds	DSP Mid Cap - Regular (D)	0.008457	0.047627	0.576107
Small Cap Funds	SBI Small Cap Fund - D (G)	0.024281	0.050545	0.600671
Small-Cap Funds	SBI Small Cap Fund (G)	0.023265	0.050543	0.60127
Small Cap Funds	Reliance Small Cap - Direct (G)	0.022386	0.053152	0.725037
Small Cap Funds	Franklin (I) Smaller Co -Direct (G)	0.018934	0.041964	0.606868
Small Cap Funds	Axis Small Cap Fund - Direct (G)	0.018092	0.040727	0.549612
Small Cap Funds	Reliance Small Cap Fund (G)	0.021501	0.053116	0.724158
Small Cap Funds	DSP Small Cap Fund - Direct (G)	0.019976	0.049643	0.69403
Small Cap Funds	DSP Small Cap Fund - Direct (D)	0.019973	0.049637	0.693933

Small Cap Funds	DSP Small Cap Fund - Regular (G)	0.019468	0.049582	0.693587
Small Cap Funds	DSP Small Cap Fund - Regular (D)	0.019469	0.049584	0.693585
Small Cap Funds	HSBC Small Cap Equity Fund - Direct (G)	0.017159	0.052693	0.835492
Small Cap Funds	HSBC Small Cap Equity Fund (G)	0.016535	0.05266	0.834984
Small Cap Funds	SBI Small Cap Fund - D (D)	0.018379	0.062259	0.663595
Small Cap Funds	Reliance Small Cap - Direct (D)	0.017052	0.058628	0.789795
Small Cap Funds	Kotak Small Cap Fund - D (D)	0.013841	0.043145	0.600917
Small Cap Funds	Reliance Small Cap Fund (D)	0.01601	0.058787	0.791523
Small Cap Funds	HSBC Small Cap Equity Fund - Direct (D)	0.015045	0.057491	0.884842
Small Cap Funds	SBI Small Cap Fund (D)	0.015066	0.064483	0.661169
Small Cap Funds	Quant Small Cap Fund - D (G)	0.006053	0.008307	0.079899
Small Cap Funds	Quant Small Cap Fund (G)	0.005979	0.008349	0.08022
Small Cap Funds	ICICI Pru Smallcap Fund - RP (D)	0.004276	0.046816	0.641856
Small Cap Funds	Quant Small Cap Fund - D (D)	0.003632	0.009746	0.05251

Source: authors calculation