# COMPARISON STUDY BETWEEN GROUP AND INVIDUAL REPLACEMENT UNDER UNCERTAIN ENVIRONMENT

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**ABSTRACT:**

In this "study we address the group replacement policy in hazy and unclear situations, such as when the item can no longer satisfy actual needs, when new technology is available, or when the item malfunctions fatally. Some machines completely stop functioning after a certain amount of time, rather than breaking down. These kinds of issues are evaluated and handled using the group replacement theory. In this case, materials or equipment fail instantly and without previous notice. Replacing these malfunctioning products simultaneously at regular intervals is more cost-effective than doing so only when they break. When our replacement model needs more than a binary concept, the idea of fuzzy comes into play. In this study we used the trapezoidal and triangular intuitionistic fuzzy numbers to identify the best solution to the replacement problem. The triangular and the trapezoidal intuitionistic fuzzy number offer a more flexible platform for expressing inaccuracy, insufficient, and imprecise as well as for reflecting the evaluation information across multiple dimensions when solving multi-criteria decision-making problems. In this study, all of the attributes, including capital cost and maintenance cost, are represented by trapezoidal and triangular intuitionistic fuzzy numbers. The proposed methodology can handle the Group and Individual Replacement problems in this generalized platform. The proposed ranking method for intuitionistic fuzzy numbers can be used to compare the average intuitionistic fuzzy cost. As a result, we may compare the average intuitionistic fuzzy cost to decide when to replace the equipment. Two examples are solved for the proposed model.

**Keywords**:*Intuitionistic fuzzy set,Triangular intuitionistic fuzzy numbers, Trapezoidal and intuitionistic fuzzy numbers, Fuzzy Replacement.*

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## [1] INTRODUCTION

Humans' reliance on machines has grown exponentially. Regular maintenance is necessary for the health of any machine that cannot run indefinitely. Sometimes it's necessary to replace the entire setup or a component of a machine. In this work, the replacement problem is mathematically addressed. When an item fails unexpectedly, maintenance costs are too high, or a new technological development takes place, a replacement is necessary. Here, we'll talk about two different replacements. The first is an individual replacement, in which one component of a complete system is changed. Group replacement is the second. Group replacement theory is a technique used to examine failures. Many of the goods in this place are deteriorating toward the end of their typical lifespan. Although this type of equipment may not require much maintenance, it can break unexpectedly and without notice. Additionally, an immediate replacement might not be available in the event of unforeseen malfunctions. Remember that group replacement does occasionally include simultaneous replacements, with individual replacements coming in between. Light bulbs, fluorescent tubes, electronic chips, and fuses are a few examples. It has been discovered that replacing these randomly failing parts at regular intervals is more cost-effective than doing so only when an item fails. The cost of individual replacements rises when groups are replaced infrequently, while frequent group replacements are unquestionably expensive. With it comes the necessity to strike a balance and determine the ideal replacement time for the ideal replacement cost. The concept of fuzzy comes into play where our mathematical model requires more than binary concept. When solving multi-criteria decision-making problems, the triangular intuitionistic fuzzy number and the trapezoidal intuitionistic fuzzy number provide a more versatile platform for expressing erroneous, insufficient, and inconsistent information as well as for reflecting the evaluation information across multiple dimensions. Trapezoidal and triangular intuitionistic fuzzy numbers are used in this work to represent all of the characteristics, including capital cost and maintenance cost. The Group and Individual Replacement problems can be resolved in this generalized platform using the proposed methodology. The average intuitionistic fuzzy cost can be compared using the proposed ranking approach for intuitionistic fuzzy numbers which is derived using the centroid method. Therefore, we could choose when to replace the equipment by comparing the average intuitionistic fuzzy cost.

## 2. PRELIMINARIES

### 2.1 INTUITIONISTIC FUZZY NUMBER:

An intuitionistic fuzzy number $\tilde{A}^{IFN}$ is,

(i)An intuitionistic fuzzy subset of the real line,

(ii)Normal, that is there is any $a_0 \in \Re$, such that

$$\mu_{\tilde{A}IFN}(a_0)=1, \nu_{\tilde{A}IFN}(a_0)=0.$$

(iii)Convex for the membership function $\mu_{\tilde{A}IFN}(a)$, that is,

$$\mu_{\tilde{A}IFN}(\lambda a_1+(1-\lambda)a_2) \ge \min(\mu_{\tilde{A}IFN}(a_1), \mu_{\tilde{A}IFN}(a_2)) \text{ for every } a_1, a_2 \in \Re, \lambda \in [0,1].$$

(iv) Concave for the non-membership function $\nu_{\tilde{A}IFN}(a)$, that is,

$$\nu_{\tilde{A}IFN}(\lambda a_1+(1-\lambda)a_2) \ge \max(\nu_{\tilde{A}IFN}(a_1); \nu_{\tilde{A}IFN}(a_2)) \text{ for every } a_1, a_2 \in \Re, \lambda \in [0,1].$$

### 2.2 TRAPEZOIDAL INTUITIONISTIC FUZZY NUMBER:

If $\tilde{A}^{IFN}$ be an intuitionistic trapezoidal fuzzy number; its membership function and non-membership functions can be defined by

$$\mu_{\tilde{A}IFN}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1}, & \text{for } a_1 \le x \le a_2 \\ = \mu_{\tilde{A}IFN}, & \text{for } a_2 \le x \le a_3 \\ \dfrac{a_4-x}{a_4-a_3}\mu_{\tilde{A}IFN}, & \text{for } a_3 \le x \le a_4 \\ = 0, & \text{otherwise} \end{cases}$$

$$\nu_{\tilde{A}IFN}(x)=\begin{cases}\dfrac{d_2-x+\nu_{\tilde{A}IFN}(x-d_1)}{d_2-d_1}, & \text{for } d_1\le x\le d_2\\ =\nu_{\tilde{A}IFN}, & \text{for } d_2\le x\le d_3\\ \dfrac{x-d_3+\nu_{\tilde{A}IFN}(d_1-x)}{d_1-d_3}, & \text{for } d_3\le x\le d_4\\ =0, & \text{otherwise}\end{cases}$$

Where $0\le\mu_{\tilde{A}IFN}\le 1; 0\le\nu_{\tilde{A}IFN}\le 1$ and $\mu_{\tilde{A}IFN}+\nu_{\tilde{A}IFN}\le 1; a_1,a_2,a_3,a_4\in\Re$.

**2.3 ARITHMETIC OPERATIONS ON` TRAPEZOIDAL INTUIONISTIC FUZZY NUMBERS:**

For any two arbitrary trapezoidal intuitionistic fuzzy numbers parametric representationsare

$\tilde{A}^{IFN}\approx(a_\mu,a^\mu,\alpha_{\mu_a},\beta_{\mu_a};a_\nu,a^\nu,\alpha_{\nu_a},\beta_{\nu_a})$ The operations are given below,

(i) Addition:

$$\tilde{A}^{IFN}+\tilde{B},^{IFN}\approx(a_\mu+b_\mu,\max(a^\mu,b^\mu),\max(\alpha_{\mu_a},\alpha_{\mu_b}),\max(\beta_{\mu_a},\beta_{\mu_b});$$
$$a_\nu+b_\nu,\max(a^\nu,b^\nu),\max(\alpha_{\nu_a},\alpha_{\nu_b}),\max(\beta_{\nu_a},\beta_{\nu_b}))$$

(ii) Subtraction:

$$\tilde{A}^{IFN}-\tilde{B},^{IFN}\approx(a_\mu-b_\mu,\min(a^\mu,b^\mu),\min(\alpha_{\mu_a},\alpha_{\mu_b}),\min(\beta_{\mu_a},\beta_{\mu_b});$$
$$a_\nu-b_\nu,\min(a^\nu,b^\nu),\min(\alpha_{\nu_a},\alpha_{\nu_b}),\min(\beta_{\nu_a},\beta_{\nu_b}))$$

(iii) Multiplication:

$$\tilde{A}^{IFN}*\tilde{B},^{IFN}\approx(a_\mu*b_\mu,\max(a^\mu,b^\mu),\max(\alpha_{\mu_a},\alpha_{\mu_b}),\max(\beta_{\mu_a},\beta_{\mu_b});$$
$$a_\nu*b_\nu,\max(a^\nu,b^\nu),\max(\alpha_{\nu_a},\alpha_{\nu_b}),\max(\beta_{\nu_a},\beta_{\nu_b}))$$

(iv) Division:

$$\tilde{A}^{IFN}/\tilde{B}^{IFN}\approx(a_\mu/b_\mu,\min(a^\mu,b^\mu),\min(\alpha_{\mu_a},\alpha_{\mu_b}),\min(\beta_{\mu_a},\beta_{\mu_b});$$
$$a_\nu/b_\nu,\min(a^\nu,b^\nu),\min(\alpha_{\nu_a},\alpha_{\nu_b}),\min(\beta_{\nu_a},\beta_{\nu_b}))$$

(v)Scalar multiplication:

$$k\tilde{A}^{IFN}\approx(ka_\mu,\max(a^\mu,b^\mu),\max(\alpha_{\mu_a},\alpha_{\mu_b}),\max(\beta_{\mu_a},\beta_{\mu_b});$$
$$ka_\nu,\max(a^\nu,b^\nu),\max(\alpha_{\nu_a},\alpha_{\nu_b}),\max(\beta_{\nu_a},\beta_{\nu_b}))$$

**2.4 ARITHMETIC OPERATIONS ON` TRIANGULAR INTUIONISTIC FUZZY NUMBERS:**

For any two arbitrary triangular intuitionistic fuzzy numbers parametric representations are

$\tilde{A}^{IFN}\approx(a_\mu,\alpha_{\mu_a},\beta_{\mu_a};a_\nu,\alpha_{\nu_a},\beta_{\nu_a})$,

$$\tilde{B}^{IFN} \approx (b_{\mu}, \alpha_{\mu_b}, \beta_{\mu_b}; b_{\nu}, \alpha_{\nu_b}, \beta_{\nu_b})$$

The operations are given below,

(i) Addition:

$$\tilde{A}^{IFN} + \tilde{B},^{IFN} \approx (a_{\mu} + b_{\mu}, \max(\alpha_{\mu_a}, \alpha_{\mu_b}), \max(\beta_{\mu_a}, \beta_{\mu_b});$$
$$a_{\nu} + b_{\nu}, \max(\alpha_{\nu_a}, \alpha_{\nu_b}), \max(\beta_{\nu_a}, \beta_{\nu_b}))$$

(ii) Subtraction:

$$\tilde{A}^{IFN} - \tilde{B},^{IFN} \approx (a_{\mu} - b_{\mu}, \min(\alpha_{\mu_a}, \alpha_{\mu_b}), \min(\beta_{\mu_a}, \beta_{\mu_b});$$
$$a_{\nu} - b_{\nu}, \min(\alpha_{\nu_a}, \alpha_{\nu_b}), \min(\beta_{\nu_a}, \beta_{\nu_b}))$$

(iii) Multiplication:

$$\tilde{A}^{IFN} * \tilde{B},^{IFN} \approx (a_{\mu} * b_{\mu}, \max(a^{\mu}, b^{\mu}), \max(\alpha_{\mu_a}, \alpha_{\mu_b}), \max(\beta_{\mu_a}, \beta_{\mu_b});$$
$$a_{\nu} * b_{\nu}, \max(a^{\nu}, b^{\nu}), \max(\alpha_{\nu_a}, \alpha_{\nu_b}), \max(\beta_{\nu_a}, \beta_{\nu_b}))$$

(iv) Division:

$$\tilde{A}^{IFN} / \tilde{B}^{IFN} \approx (a_{\mu} / b_{\mu}, \min(a^{\mu}, b^{\mu}), \min(\alpha_{\mu_a}, \alpha_{\mu_b}), \min(\beta_{\mu_a}, \beta_{\mu_b});$$
$$a_{\nu} / b_{\nu}, \min(a^{\nu}, b^{\nu}), \min(\alpha_{\nu_a}, \alpha_{\nu_b}), \min(\beta_{\nu_a}, \beta_{\nu_b}))$$

(v)Scalar multiplication:

$$k\tilde{A}^{IFN} \approx (ka_{\mu}, \max(a^{\mu}, b^{\mu}), \max(\alpha_{\mu_a}, \alpha_{\mu_b}), \max(\beta_{\mu_a}, \beta_{\mu_b});$$
$$ka_{\nu}, \max(a^{\nu}, b^{\nu}), \max(\alpha_{\nu_a}, \alpha_{\nu_b}), \max(\beta_{\nu_a}, \beta_{\nu_b}))$$

## 3. THE PROPOSED MODEL

### 3.1 GROUP REPLACEMENT POLICY OF ITEMS THAT FAIL SUDDENLY AND COMPLETELY

Let

$\tilde{N}$ = Total number of items in the system.

$\tilde{N}(x)$ =Number of items failed during the x^th period

$\tilde{\zeta}_g$ = Fuzzy group replacement cost per item.

$\tilde{\zeta}_i$ =Individual replacement fuzzy cost per item.

$$\tilde{\zeta}(n) = \tilde{N}\,\tilde{\zeta}_g + \tilde{\zeta}_i[\tilde{N}(1) + \tilde{N}(2) + \dots\dots\dots + \tilde{N}(n-1)] = \tilde{N}\,\tilde{\zeta}_g + \tilde{\zeta}_i \sum_{x=1}^{(n-1)} \tilde{N}(x)$$

Average fuzzy cost per period = $\tilde{A}(n) = \dfrac{\tilde{\zeta}(n)}{n}$

Since n is a discrete variable,

$\tilde{A}(n)$ is minimum

$[\Delta\tilde{A}(n-1) < 0 < \Delta\tilde{A}(n)]$

$$\Delta\tilde{A}(n) = \tilde{A}(n+1) - \tilde{A}(n) = \frac{\tilde{\zeta}(n+1)}{n+1} - \frac{\tilde{\zeta}(n)}{n}$$

From $\tilde{\zeta}(n)$, we get $\tilde{\zeta}(n+1) = \tilde{\zeta}(n) + \tilde{\zeta}_i\, \tilde{N}(n)$



$$\Delta\tilde{A}(n) = \frac{\tilde{\zeta}(n) + \tilde{\zeta}_i \tilde{N}(n)}{(n+1)} - \frac{\tilde{\zeta}(n)}{(n)}$$

$$\Delta\tilde{A}(n) = \frac{\tilde{\zeta}_i \tilde{N}(n) - \frac{\tilde{\zeta}(n)}{(n)}}{(n+1)}$$

$$\Delta\tilde{A}(n-1) = \frac{\tilde{\zeta}_i \tilde{N}(n-1) - \frac{\tilde{\zeta}(n-1)}{(n-1)}}{(n)}$$

Similarly we get

$$\tilde{\zeta}_i \tilde{N}(n-1) - \frac{\tilde{\zeta}(n-1)}{(n-1)} < 0 < \tilde{\zeta}_i \tilde{N}(n) - \frac{\tilde{\zeta}(n)}{(n)}$$

$$\tilde{\zeta}_i \tilde{N}(n) > \frac{\tilde{\zeta}(n)}{(n)}$$

$$\tilde{\zeta}_i \tilde{N}(n-1) < \frac{\tilde{\zeta}(n-1)}{(n-1)}$$

**3.2 Theorem:**

(a) Group replacement has to be made at the end of$t^{th}$ period if the cost of individual replacements for the period t is more than the average fuzzy cost per period through the end of the period, t.

(b) Group replacement is not advisable at the end of period t, if the cost of individual replacement at the end of period t–1 is less than the average fuzzy cost per period through the end of $i^{th}$ period.

Proof:

Let, $\tilde{N}$ = Total number of items in the system.

$\tilde{N}_t$ = Number of items failing during time t.

$\tilde{\zeta}(t)$ = Total fuzzy cost of group replacement after time period t.

$\tilde{\zeta}_i$ = Individual fuzzy replacement cost on failure.

$\tilde{\zeta}_g$ = Per item fuzzy cost of group replacement.

$$\tilde{\zeta}(t) = \tilde{\zeta}_i[\tilde{N}_1 + \tilde{N}_2 + \tilde{N}_3 + .... + \tilde{N}_{t-1}] + \tilde{\zeta}_g \tilde{N}$$

Hence

The average fuzzy cost of group replacement will be denoted by $\tilde{Z}(t)$

$$\tilde{Z}(t)=\frac{\tilde{\zeta}(t)}{t}=\frac{\tilde{\zeta}_i[\tilde{N}_1+\tilde{N}_2+\tilde{N}_3+....+\tilde{N}_{t-1}]+\tilde{\zeta}_g\tilde{N}}{t}$$

The condition for minimum of $\tilde{Z}(t)$ is,

$$\tilde{Z}(t)=\tilde{Z}(t+1)-\tilde{Z}(t)=\frac{\tilde{\zeta}(t+1)}{t+1}-\frac{\tilde{\zeta}(t)}{t}=\frac{\tilde{\zeta}(t)+\tilde{\zeta}_i(\tilde{N}_t)}{t+1}-\frac{\tilde{\zeta}(t)}{t}$$



$$\frac{t\tilde{\zeta}_i\tilde{N}_t-\tilde{\zeta}(t)}{t(t+1)}=\frac{\tilde{\zeta}_i\tilde{N}_t-\tilde{\zeta}(t)/t}{t(t+1)}$$

Which should be greater than 0 for minimum $\tilde{Z}(t)$.

$\tilde{\zeta}_i\tilde{N}_t>\tilde{\zeta}(t)/t$

When $\Delta\tilde{Z}(t-1)<0$

We get $\tilde{\zeta}_i\tilde{N}_{t-1}>\tilde{\zeta}(t)/t$

Hence $\tilde{\zeta}_i\tilde{N}_{t-1}<\tilde{\zeta}(t)/t<\tilde{\zeta}_i\tilde{N}_t$

It can also be written as $\tilde{\zeta}_i\tilde{N}_t>\tilde{\zeta}(t)/t\Rightarrow(\tilde{\zeta}_i\tilde{N}_t/t+1)>(\tilde{\zeta}(t)/t(t+1))$

$$\Rightarrow\frac{\tilde{\zeta}_i\tilde{N}_t}{t+1}+\frac{\tilde{\zeta}(t)}{t+1}>\frac{\tilde{\zeta}(t)}{t(t+1)}+\frac{\tilde{\zeta}(t)}{t+1}$$

$$\Rightarrow\frac{\tilde{\zeta}_i\tilde{N}_t+\tilde{\zeta}(t)}{t+1}>\frac{\tilde{\zeta}(t)}{t+1}\left[1+\frac{1}{t}\right]\Rightarrow\frac{\tilde{\zeta}(t+1)}{t+1}>\frac{\tilde{\zeta}(t)}{t}$$

It can be written as

The average fuzzy cost for (t+1) periods >average fuzzy cost for t periods.

Table: Failure rates of items

| period | 1 | 2 | 3 | 4 | 5 | ………… | K |
|---|---|---|---|---|---|---|---|
| Failure probability | $\delta_1$ | $\delta_2$ | $\delta_3$ | $\delta_4$ | $\delta_5$ | ………… | $\delta_k$ |

Let $\rho_i$ is the probability of failure in $i^{th}$ period

$\rho_1=\delta_1$

$\rho_2=\delta_2-\delta_1$

$\rho_3=\delta_3-\delta_2$

$\rho_4=\delta_4-\delta_3$

$\rho_5=\delta_5-\delta_4$

……………………………………

$\rho_k=\delta_k-\delta_{k-1}$

Since the"sum of probabilities can never be greater than 1, and when the sum of all the above probabilities, say up to 5th period is 1, the further probabilities $\rho_6,\rho_7,\rho_8$ …so on", will be zero. Thus, all items are sure to fail by the end of $5^{th}$ week.

Let, $\tilde{N}_i$ = the number of replacements at the end of the i$^{th}$ period,

Thus,

$$\tilde{N}_0 = \tilde{N}_0$$
$$\tilde{N}_1 = \tilde{N}_0 \rho_1$$
$$\tilde{N}_2 = \tilde{N}_0 \rho_2 + \tilde{N}_1 \rho_1$$
$$\tilde{N}_3 = \tilde{N}_0 \rho_3 + \tilde{N}_1 \rho_2 + \tilde{N}_2 \rho_1$$
$$\tilde{N}_4 = \tilde{N}_0 \rho_4 + \tilde{N}_1 \rho_3 + \tilde{N}_2 \rho_2 + \tilde{N}_3 \rho_1$$

...........

And so on up to k$^{th}$ period

It has been studied that expected number of items failing in each period increased initially and then decreased.

**Individual Replacement:**

The average age of item = $1X\rho_1 + 2X\rho_2 + 3X\rho_3 + 4X\rho_4 + 5X\rho_5 + .... + kX\rho_k = \sum_{i=1}^{n} i\rho_i$

The number of failures in each period in steady state = $\tilde{N} / \sum_{i=1}^{n} i\rho_i$

The cost of replacing items individually on failure = $\left[ \tilde{N} / \sum_{i=1}^{n} i\rho_i \right] \tilde{\zeta}_i$

**Group Replacement:**

The replacement of all N items simultaneously costs $\tilde{C}_g$ per item and replacement of an individual item on failure costs $\tilde{C}_i$ .

It would be optimal to group replace all the items after every i$^{th}$period, when the fuzzy cost of individual replacement in the (i+1)$^{th}$ period is greater than the average fuzzy cost for i$^{th}$ period.

| End of period | Number of individual replacement in each month | Total cost of group replacement Average cost | Average cost per period |
|---|---|---|---|
| 1 | $\tilde{N}_1$ | $\tilde{p}_1(\tilde{N}_1) + \tilde{p}_2\tilde{N}_0$ | $(\tilde{p}_1(\tilde{N}_1) + \tilde{p}_2\tilde{N}_0)/1$ |
| 2 | $\tilde{N}_1 + \tilde{N}_2$ | $\tilde{p}_1(\sum_{i=1}^{2}\tilde{N}_i) + \tilde{p}_2\tilde{N}_0$ | $(\tilde{p}_1(\sum_{i=1}^{2}\tilde{N}_i) + \tilde{p}_2\tilde{N}_0)/2$ |
| 3 | $\tilde{N}_1 + \tilde{N}_2 + \tilde{N}_3$ | $\tilde{p}_1(\sum_{i=1}^{3}\tilde{N}_i) + \tilde{p}_2\tilde{N}_0$ | $(\tilde{p}_1(\sum_{i=1}^{3}\tilde{N}_i) + \tilde{p}_2\tilde{N}_0)/3$ |
| 4 | $\tilde{N}_1 + \tilde{N}_2 + \tilde{N}_3 + \tilde{N}_4$ | $\tilde{p}_1(\sum_{i=1}^{4}\tilde{N}_i) + \tilde{p}_2\tilde{N}_0$ | $(\tilde{p}_1(\sum_{i=1}^{4}\tilde{N}_i) + \tilde{p}_2\tilde{N}_0)/4$ |

Further, when the individual replacement cost after i$^{th}$ period is less than the group replacement cost at the end of i$^{th}$ period the individual replacement after i$^{th}$ period is preferable.

**4. RANKING OF INTUITIONISTIC FUZZY NUMBER BASED ON CENTROID CONCEPT:**

In this study, we calculate the centroid point of the trapezoidal and triangular intuitionistic fuzzy numbers. Consider a trapezoidal intuitionistic fuzzy number $\tilde{A}$ ,its membership function and the non membership function are definedby

$$\mu_{\tilde{A}} = \begin{cases} 0 & a_1 > x \\ f_A^L(x), & a_1 \le x \le a_2 \\ 1, & a_2 \le x \le a_3 \\ f_A^R(x), & a_3 \le x \le a_4 \\ 0, & a_4 \le x \end{cases}$$

$$\nu_{\tilde{A}} = \begin{cases} 0 & d_1 > x \\ g_{\tilde{A}}^L(x) & d_1 \le x \le d_2 \\ 0 & d_2 \le x \le d_3 \\ g_{\tilde{A}}^R(x) & d_3 \le x \le d_4 \\ 1 & d_4 < x \end{cases}$$

Here, $f_{\tilde{A}}^L : \mathrm{R} \to [0,1]$ , $g_{\tilde{A}}^R : \mathrm{R} \to [0,1]$ are non decreasing functions $f_{\tilde{A}}^R : \mathrm{R} \to [0,1]$ $g_{\tilde{A}}^L : \mathrm{R} \to [0,1]$ are non increasing functions. The centroid point of the trapezoidal intuitionistic fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4; d_1, d_2, d_3, d_4)$ can be written as

$$\tilde{z}_\mu(\tilde{A}) = \frac{\int_{a_1}^{a_2} \frac{z^2 - za_1}{\alpha_\mu} dz + \int_{a_2}^{a_3} z dz - \int_{a_3}^{a_4} \frac{z^2 - za_1}{\beta_\mu} dz}{\int_{a_1}^{a_2} \frac{z^2 - a_1}{\alpha_\mu} dz + \int_{a_2}^{a_3} dz - \int_{a_3}^{a_4} \frac{z^2 - a_1}{\beta_\mu} dz}$$

$$\tilde{z}_\mu(\tilde{A}) = \frac{\frac{1}{\alpha_\mu}\left[\frac{2(a_2^3 - a_1^3) - 3(a_2^2 - a_1^2)a_1}{6}\right] + \frac{(a_3^2 - a_2^2)}{2} - \frac{1}{\beta_\mu}\left[\frac{2(a_4^3 - a_3^3) - 3(a_4^2 - a_3^2)a_4}{6}\right]}{\frac{1}{\alpha_\mu}\left[\frac{(a_2^2 - a_1^2)}{2} - (a_2 - a_1)a_1\right] + (a_3 - a_2) + \frac{1}{\beta_\mu}\left[\frac{(a_4^2 - a_3^2)}{2} - (a_4 - a_3)a_4\right]}$$

$$\tilde{z}_{\mu}(\tilde{A}) = \frac{\frac{1}{\alpha_{\mu}}\left[\frac{2(\alpha_{\mu}^{3}) + 6a_2a_1\alpha_{\mu} - 3\alpha_{\mu}(a_2a_1 + a_1^2)}{6}\right] + \frac{(a_3^2 - a_2^2)}{2} - \frac{1}{\beta_{\mu}}\left[\frac{2(\beta_{\mu}^{3}) + 6a_4a_3\beta_{\mu} - 3\beta_{\mu}(a_3a_4 + a_4^2)}{6}\right]}{\frac{1}{\alpha_{\mu}}\left[\frac{\alpha_{\mu}(a_2 + a_1)}{2} - \alpha_{\mu}a_1\right] + (a_3 - a_2) - \frac{1}{\beta_{\mu}}\left[\frac{\beta_{\mu}(a_4 + a_3)}{2} - \beta_{\mu}a_4\right]}$$

$$\tilde{z}_{\mu} = \frac{1}{3}\left[\frac{2\alpha_{\mu}^2 + 3a_2a_1 - 3a_1^2 + 3(a_3^2 - a_2^2) - 2\beta_{\mu}^2 - 3a_4a_3 + 3a_4^2}{a_4 + a_3 - a_2 - a_1}\right]$$

Where $\alpha_{\mu}$=left spread and $\beta_{\mu}$=Right spread of membership function

$$\tilde{z}_{\nu}(\tilde{A}) = \frac{\int_{d_1}^{d_2} \frac{z^2 - zd_2}{-\alpha_{\nu}}\,dx + \int_{d_2}^{d_3} z\,dz + \int_{d_3}^{d_4} \frac{z^2 - zd_3}{\beta_{\nu}}\,dz}{\int_{d_1}^{d_2} \frac{z^2 - d_2}{-\alpha_{\nu}}\,dz + \int_{d_2}^{d_3} dz + \int_{d_3}^{d_4} \frac{z^2 - d_3}{\beta_{\nu}}\,dz}$$

$$\tilde{z}_{\nu}(\tilde{A}) = \frac{-\frac{1}{\alpha_{\nu}}\left[\frac{2(d_2^3 - d_1^3) - 3(d_2^2 - d_1^2)d_2}{6}\right] + \frac{(d_3^2 - d_2^2)}{2} + \frac{1}{\beta_{\nu}}\left[\frac{2(d_4^3 - d_3^3) - 3(d_4^2 - d_3^2)d_3}{6}\right]}{-\frac{1}{\alpha_{\nu}}\left[\frac{(d_2^2 - d_1^2)}{2} - d_2\alpha_{\nu}\right] + (d_3 - d_2) + \frac{1}{\beta_{\nu}}\left[\frac{(d_4^2 - d_3^2)}{2} - d_3\beta_{\nu}\right]}$$

$$\tilde{z}_{\nu}(\tilde{A}) = \frac{-\frac{1}{\alpha_{\nu}}\left[\frac{2(\alpha_{\nu}^{3}) + 6d_2d_1\alpha_{\nu} - 3\alpha_{\nu}(d_2d_1 + d_1^2)}{6}\right] + \frac{(d_3^2 - d_2^2)}{2} + \frac{1}{\beta_{\nu}}\left[\frac{2(\beta_{\nu}^{3}) + 6d_4d_3\beta_{\nu} - 3\beta_{\nu}(d_3d_4 + d_4^2)}{6}\right]}{-\frac{1}{\alpha_{\nu}}\left[\frac{\alpha_{\nu}(d_2 + d_1)}{2} - \alpha_{\nu}d_2\right] + (d_3 - d_2) + \frac{1}{\beta_{\nu}}\left[\frac{\beta_{\nu}(d_4 + d_3)}{2} - \beta_{\nu}d_3\right]}$$

$$\tilde{z}_{\nu} = \frac{1}{3}\left[\frac{-2\alpha_{\nu}^{2} - 3d_{2}d_{1} + 3d_{1}^{2} + 3(d_{3}^{2} - d_{2}^{2}) + 2\beta_{\nu}^{2} + 3d_{4}d_{3} - 3d_{3}^{2}}{d_{4} + d_{3} - d_{2} - d_{1}}\right]$$



Where $\alpha_{\nu}$ =left spread and $\beta_{\nu}$ =Right spread of Non membership function

$$\tilde{w}_{\mu}(\tilde{A}) = \frac{\int_{0}^{1}((\alpha_{\mu})w^{2} + a_{1}w)dw - \int_{0}^{1}((-\beta_{\mu})w^{2} + a_{4}w)dw}{\int_{0}^{1}((\alpha_{\mu})w + a_{1})dw - \int_{0}^{1}((-\beta_{\mu})w + a_{4})dw}$$

$$\tilde{w}_{\mu} = \frac{\dfrac{2\alpha_{\mu} + 3a_{1} - 3a_{4} + 2\beta_{\mu}}{6}}{\dfrac{\alpha_{\mu}}{2} + a_{1} + \dfrac{\beta_{\mu}}{2} - a_{4}}$$

$$\tilde{w}_{\mu} = \frac{1}{3}\left[\frac{2\alpha_{\mu} + 3a_{1} - 3a_{4} + 2\beta_{\mu}}{\alpha_{\mu} + 2a_{1} - 2a_{4} + \beta_{\mu}}\right]$$

Where $\alpha_{\mu}$ =left spread and $\beta_{\mu}$ =Right spread of membership function

$$\tilde{w}_{\nu} = \frac{\int_{0}^{1}((-\alpha_{\nu})w^{2} + d_{2}w)dw - \int_{0}^{1}((\beta_{\nu})w^{2} + d_{3}w)dw}{\int_{0}^{1}((-\alpha_{\nu})w + d_{2})dw - \int_{0}^{1}((\beta_{\nu})w + d_{3})dw}$$

$$\tilde{w}_{\nu} = \frac{\frac{1}{6}\left[-2\alpha_{\nu} + 3d_{2} - 2\beta_{\nu} - 3d_{3}\right]}{-\dfrac{\alpha_{\nu}}{2} + d_{2} - \dfrac{\beta_{\nu}}{2} - d_{3}}$$

$$\tilde{w}_{\nu} = \frac{1}{3}\left[\frac{2\alpha_{\nu} - 3d_{2} + 3d_{3} + 2\beta_{\nu}}{\alpha_{\nu} - 2d_{2} + 2d_{3} + \beta_{\nu}}\right]$$

Where $\alpha_{\nu}$ =left spread and $\beta_{\nu}$ =Right spread of Non membership function. $\tilde{z}_{\mu}, \tilde{z}_{\nu}, \tilde{w}_{\mu}, \tilde{w}_{\nu}$ are the points of the centroid.

Hence ranking of trapezoidal(triangular) intuitionistic fuzzy number is defined in case (i&ii)

Case(i):The ranking method can be used to compare the two trapezoidal intuitionistic fuzzy number.

$$R(\tilde{A}^{IFN}) = \sqrt{\frac{1}{2}\left(\left[\tilde{z}_{\mu}(\tilde{A}) - \tilde{w}_{\mu}(\tilde{A})\right]^{2} + \left[\tilde{z}_{\nu}(\tilde{A}) - \tilde{w}_{\nu}(\tilde{A})\right]^{2}\right)}$$

Where

$$\tilde{z}_{\mu}=\frac{1}{3}\left[\frac{2\alpha_{\mu}^{2}+3a_{2}a_{1}-3a_{1}^{2}+3(a_{3}^{2}-a_{2}^{2}) -2\beta_{\mu}^{2}-3a_{4}a_{3}+3a_{4}^{2}}{a_{4}+a_{3}-a_{2}-a_{1}}\right]$$



$$\tilde{z}_{\nu}=\frac{1}{3}\left[\frac{-2\alpha_{\nu}^{2}-3d_{2}d_{1}+3d_{2}^{2}+3(d_{3}^{2}-d_{2}^{2}) +2\beta_{\nu}^{2}+3d_{4}d_{3}-3d_{3}^{2}}{d_{4}+d_{3}-d_{2}-d_{1}}\right]$$

$$\tilde{w}_{\mu}=\frac{1}{3}\left[\frac{2\alpha_{\mu}+3a_{1}-3a_{4}+2\beta_{\mu}}{\alpha_{\mu}+2a_{1}-2a_{4}+\beta_{\mu}}\right]$$

$$\tilde{w}_{\nu}=\frac{1}{3}\left[\frac{2\alpha_{\nu}-3d_{2}+3d_{3}+2\beta_{\nu}}{\alpha_{\nu}-2d_{2}+2d_{3}+\beta_{\nu}}\right]$$

Which represents the centroid of the TraIFN

The ranking can be define by

$(i)\tilde{A}^{IFN}\succ\tilde{B}^{IFN}\Leftrightarrow R(\tilde{A}^{IFN})\succ R(\tilde{B}^{IFN})$

$(ii)\tilde{A}^{IFN}\prec\tilde{B}^{IFN}\Leftrightarrow R(\tilde{A}^{IFN})\prec R(\tilde{B}^{IFN})$

$(\text{iii})\tilde{A}^{IFN}\approx\tilde{B}^{IFN}\Leftrightarrow R(\tilde{A}^{IFN})\approx R(\tilde{B}^{IFN})$

Case(ii):The above ranking can be used to compare the triangular intuitionistic fuzzy number alsoif we take ($b_{2}=a_{2}=a_{3}=b_{3}$).

Sub $a_{3}=a_{2}$ in the above $\tilde{z}_{\mu}$

$$\tilde{z}_{\mu}=\frac{1}{3}\left[\frac{2(a_{2}-a_{1})^{2}+3a_{2}a_{1}-3a_{1}^{2}+3(a_{2}^{2}-a_{2}^{2}) -2(a_{4}-a_{2})^{2}-3a_{4}a_{2}+3a_{4}^{2}}{a_{4}+a_{2}-a_{2}-a_{1}}\right]$$

$$\tilde{z}_{\mu}=\frac{1}{3}\left[\frac{(a_{4}+a_{1})(a_{4}-a_{1})}{(a_{4}-a_{1})}+\frac{a_{2}(a_{4}-a_{1})}{(a_{4}-a_{1})}\right]$$

$$\tilde{z}_{\mu}=\left[\frac{(a_{4}+a_{1}+a_{2})}{3}\right]$$

Sub $d_{3}=d_{2}$ in $\tilde{z}_{\nu}$



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$$\tilde{z}_V = \frac{1}{3}\left[\frac{\begin{array}{l}-2d_1^2 - 2d_2^2 + 4d_2 d_1 - 3d_2 d_1 + 3d_2^2 \\ +2d_4^2 + 2d_2^2 - 4d_4 d_2 + 3d_4 d_2 - 3d_2^2\end{array}}{d_4 + d_2 - d_2 - d_1}\right]$$



$$\tilde{z}_V = \left[\frac{(d_1 - \alpha_V + 2d_4)}{3}\right]$$

Similarly

Sub $a_3 = a_2$ in $\tilde{w}_\mu$

$$\tilde{w}_\mu = \frac{1}{3}\left[\frac{\begin{array}{l}2a_2 - 2a_1 + 3a_1 - 3a_4 \\ +2a_4 - 2a_2\end{array}}{\begin{array}{l}a_2 - a_1 + 2a_1 - 2a_4 \\ +a_4 - a_2\end{array}}\right]$$

$$\tilde{w}_\mu = \frac{1}{3}\left[\frac{(a_1 - a_4)}{(a_1 - a_4)}\right] = \frac{1}{3}$$

Sub $a_3 = a_2$ in $\tilde{w}_V$

$$\tilde{w}_V = \frac{1}{3}\left[\frac{\begin{array}{l}2(d_2 - d_1) - 3d_2 \\ +3d_2 + 2d_4 - 2d_2\end{array}}{\begin{array}{l}d_2 - d_1 - 2d_2 + 2d_2 \\ +d_4 - d_2\end{array}}\right]$$

$$\tilde{w}_V = \frac{1}{3}\left[\frac{2(d_4 - d_1)}{(d_4 - d_1)}\right] = \frac{2}{3}$$

## 5.APPLICATION OF RANKING IN TRAPEZOIDAL INTUITIONISTIC FUZZY NUMBER:

### 5.1 EXAMPLE:

The failure rates of certain items are observed as above:

| End of period | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Failure probability | 0.2 | 0.3 | 0.6 | 0.85 | 1.0 |

The cost per item of individual replacement is (1.20, 1.23, 1.27, 1.30; 1.10, 1.23, 1.27, 1.40). The cost of group replacement is (0.45, 0.50, 0.55, 0.60; 0.40, 0.50, 0.55, 0.65) per item, Determine the optimal interval of replacement as whole group. Also find whether individual replacement is preferable than group replacement.

Solution:

$\rho_1 = 0.20$

$\rho_2 = 0.30 - 0.20 = 0.10$

$\rho_3 = 0.60 - 0.30 = 0.30$

$\rho_4 = 0.85 - 0.60 = 0.25$

$\rho_5 = 1.0 - 0.85 = 0.15$



Let $\tilde{N}_i$ items be replaced at i[th] month end.

$\tilde{N}$ - Number of items at the beginning.

$\tilde{N} = \tilde{N}_0 = (90, 95, 100, 105; 85, 95, 100, 110)$

The parametric form of $\tilde{N}_0$

$\tilde{N} = \tilde{N}_0 = (97.5, 2.5, 5, 5; 97.5, 2.5, 10, 10)$

$\tilde{N}_1 = \tilde{N}_0\rho_1 = (19.5, 2.5, 5, 5; 19.5, 2.5, 10, 10)$

$\tilde{N}_2 = \tilde{N}_0\rho_2 + \tilde{N}_1\rho_1 = (13.65, 2.5, 5, 5; 13.65, 2.5, 10, 10)$

$\tilde{N}_3 = \tilde{N}_0\rho_3 + \tilde{N}_1\rho_2 + \tilde{N}_2\rho_1 = (33.93, 2.5, 5, 5; 33.93, 2.5, 10, 10)$

$\tilde{N}_4 = \tilde{N}_0\rho_4 + \tilde{N}_1\rho_3 + \tilde{N}_2\rho_2 + \tilde{N}_3\rho_1 = (38.37, 2.5, 5, 5; 38.37, 2.5, 10, 10)$

$\tilde{N}_5 = \tilde{N}_0\rho_5 + \tilde{N}_1\rho_4 + \tilde{N}_2\rho_3 + \tilde{N}_3\rho_2 + \tilde{N}_4\rho_1 = (34.66, 2.5, 5, 5; 34.66, 2.5, 10, 10)$

The expected life of any item $\sum_{i=1}^{5} i\rho_i$ =3.05

The average number of failures per month = (31.96,2.5,5,5;31.96,2.5,10,10).

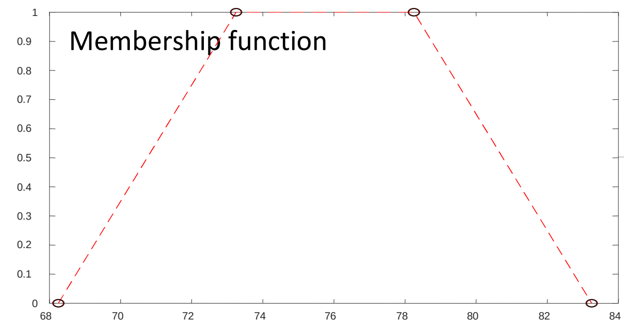
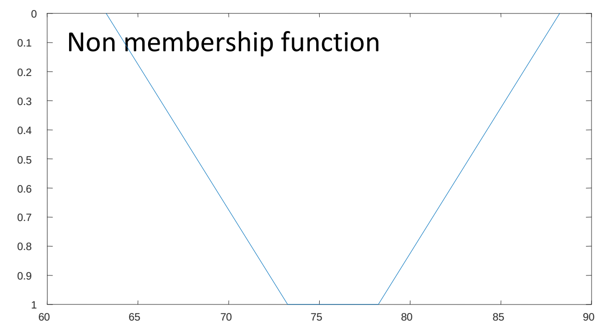
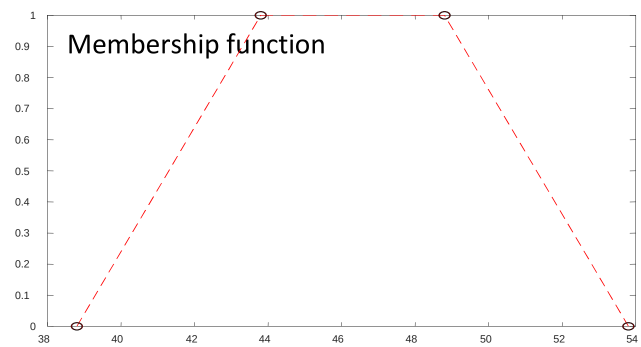
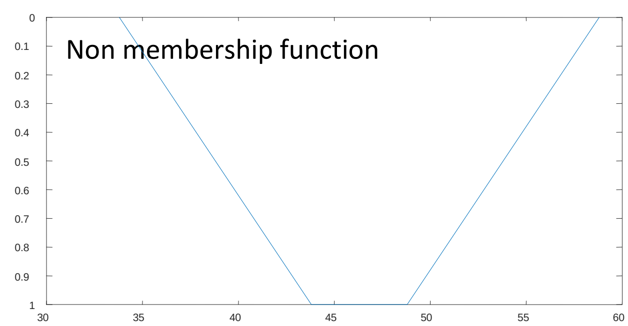
(1.25,0.02,0.03,0.03;1.25,0.02,0.13,0.13)

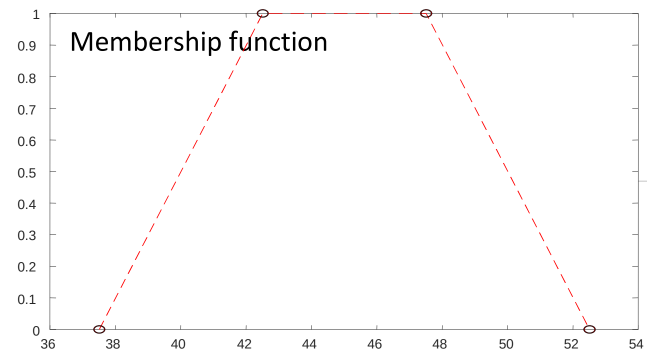
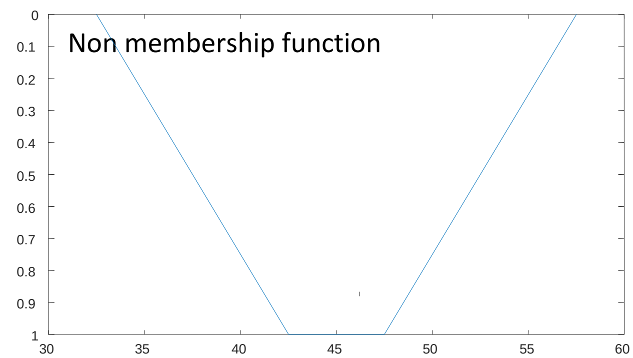
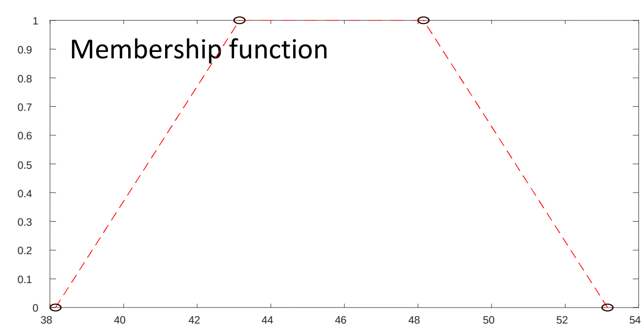
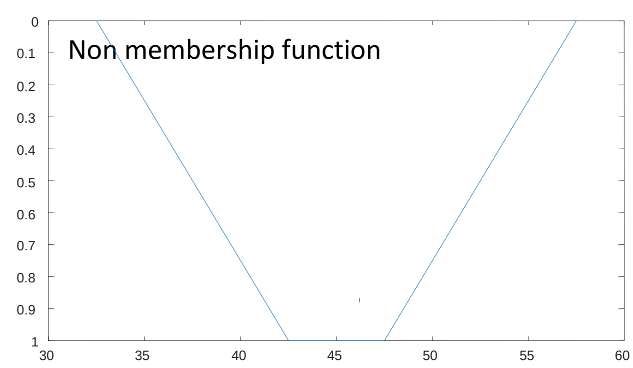
Average cost of individual replacement =(39.95,2.5,5,5;39.95,2.5,10,10)=30.36

Table :1,2Average fuzzy cost of group replacement policy

| End of the month ‘t’ | Individual Replacement intutionistic fuzzy cost $\sum_{i=1}^{t} \tilde{N}_i$ | Group Replacement intutionistic fuzzy cost $\tilde{p}_1(\sum_{i=1}^{t} \tilde{N}_i) + \tilde{p}_2\tilde{N}$ |
|---|---|---|
| 1 | (19.5,2.5,5,5;19.5,2.5,10,10) | (75.55,2.5,5,5;75.55,2.5,10,10) |
| 2 | (33.15,2.5,5,5;33.15,2.5,10,10) | (92.61,2.5,5,5;92.61,2.5,10,10) |
| 3 | (67.08,2.5,5,5;67.08,2.5,10,10) | (135.03,2.5,5,5;135.03,2.5,10,10) |
| 4 | (105.45,2.5,5,5;105.45,2.5,10,10) | (182.61,2.5,5,5;182.61,2.5,10,10) |

| End of the month ‘t’ | Average fuzzy cost= $\tilde{p}_1(\sum_{i=1}^{t} \tilde{N}_i) + \tilde{p}_2\tilde{N}$ | Graphical representation of average fuzzy cost |
|---|---|---|

| | | |
|---|---|---|
| 1 | (75.55,2.5,5,5;75.55,2.5,10,10) |     |
| 2 | (46.305,2.5,5,5;46.305,2.5,10,10) |     |
| 3 | (45.01,2.5,5,5;45.01,2.5,10,10) | |

| | | |
|---|---|---|
| | |     |
| 4 | (45.65,2.5,5,5;45.65,2.5,10,10) |     |

| End of the month ‘t’ | Ranking for comparison of the results |
|---|---|
| 1 | 75.02 |
| 2 | 45.57 |
| 3 | 44.28* |
| 4 | 44.92 |

Here the mean cost is minimum in the 3rd month, hence we have to do group replacement at the end of every 3rd month. Also, the mean cost is $ 30.36 (The mean cost in the case of individual replacement), the individual replacement policy is preferable.

Table : 3Comparison Analysis of the Results Obtained by the Ranking Method and Other Methods (to obtain the time of replacement):

| MONTH | Ranking of TraIFNs Based on Improved Accuracy Function(R. Sathya Bama) | New Approach for Ranking of Intuitionistic Fuzzy Numbers(Suresh Mohan) |
|---|---|---|
| 1 | 5.6951 | 75.75 |
| 2 | 2.1307 | 46.30 |
| 3 | 2.0142* | 45.01* |
| 4 | 2.0716 | 45.65 |

From the above comparison table and the line graph we can find that the above existing methods provided here shows that the intuitionistic fuzzy average cost which is minimum in the 3rd month. Hence the item has to be replaced at the end of 3rd month.

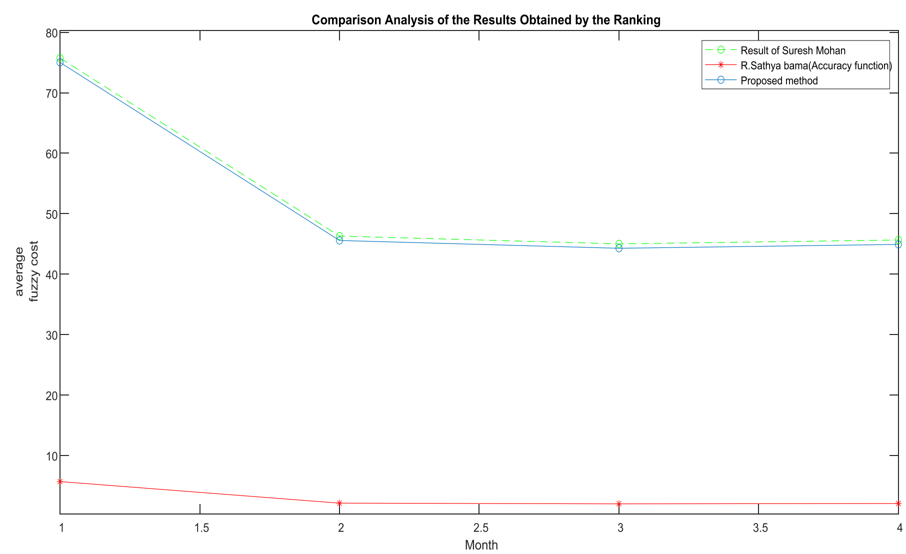



Figure 1: Comparison of results

## 2. EXAMPLE

| End of period | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Failure probability | 0.2 | 0.3 | 0.6 | 0.85 | 1.0 |



The failure rates of certain items are observed as above:The cost per item of individual replacement is (1.20,1.25,1.30;1.10,1.25,1.40). The cost of group replacement is (0.45,0.50,0.60;0.40,0.50,0.65) per item, Determine the optimal interval of replacement as whole group. Also find whether individual replacement is preferable than group replacement.

Solution:

$\rho_1 = 0.20$

$\rho_2 = 0.40 - 0.30 = 0.10$

$\rho_3 = 0.70 - 0.40 = 0.30$

$\rho_4 = 0.85 - 0.75 = 0.25$

$\rho_5 = 1.0 - 0.85 = 0.15$

Let $\tilde{N}_i$ items be replaced at i[th] month end.

$\tilde{N}$ - Number of items at the beginning.

$\tilde{N} = \tilde{N}_0 = (90,100,105;85,100,110)$

The parametric form of $\tilde{N}_0$

$\tilde{N} = \tilde{N}_0 = (100,10,5;100,15,10)$

$\tilde{N}_1 = \tilde{N}_0\rho_1 = (20,10,5;20,15,10)$

$\tilde{N}_2 = \tilde{N}_0\rho_2 + \tilde{N}_1\rho_1 = (14,10,5;14,15,10)$

$\tilde{N}_3 = \tilde{N}_0\rho_3 + \tilde{N}_1\rho_2 + \tilde{N}_2\rho_1 = (34.8,10,5;34.8,15,10)$

$\tilde{N}_4 = \tilde{N}_0\rho_4 + \tilde{N}_1\rho_3 + \tilde{N}_2\rho_2 + \tilde{N}_3\rho_1 = (39.36,10,5;39.36,15,10)$

$\tilde{N}_5 = \tilde{N}_0\rho_5 + \tilde{N}_1\rho_4 + \tilde{N}_2\rho_3 + \tilde{N}_3\rho_2 + \tilde{N}_4\rho_1 = (35.55,10,5;35.55,15,10)$

The expected life of any item $\sum_{i=1}^{5} i\rho_i$ =3.05

The average number of failures per month = (32.78, 10, 5; 32.78, 15, 10)(1.25, 0.05, 0.05; 1.25, 0.15, 0.15)Average cost of individual replacement = (40.97, 10, 5; 40.97, 15, 10)

Table :4Average cost of group replacement policy

| End of the month 't' | Individual Replacement $\sum_{i=1}^{t}\tilde{N}_i$ | Group Replacement fuzzy cost $\tilde{p}_1(\sum_{i=1}^{t}\tilde{N}_i) + \tilde{p}_2\tilde{N}$ |
|---|---|---|
| 1 | (20,10,5;20,15,10) | (75,10,5;75,15,10) |
| 2 | (34,10,5;34,15,10) | (92.5,10,5;92.5,15,10) |
| 3 | (68.8,10,5;68.8,15,10) | (136,10,5;136,15,10) |
| 4 | (108.16,10,5;108.16,15,10) | (185.2,10,5;185.2,15,10) |

Table :5Average fuzzy cost if group replacement

| End of the month 't' | Average fuzzy cost= $\tilde{p}_1(\sum_{i=1}^{t}\tilde{N}_i)+\tilde{p}_2\tilde{N}$ /t |
|---|---|
| 1 | (75,10,5;75,15,10) |
| 2 | (46.25,10,5;46.25,15,10) |
| 3 | (**45.33**,10,5;45.33,15,10) |
| 4 | (46.3,10,5;46.3,15,10) |

From the table – 5 we can find the optimal replacement time. Hence the group replacement can be done at the end of the 3rd month.

| Ranking: comparison the results |
|---|
| 72.00 |
| 43.26 |
| 42.34* |
| 43.31 |

Hence from the group replacement policy the items should to be replaced at the end of 3rd month. Therefore the minimum average cost is 43.26>42.34<43.31 which is occurs in the 3rd month and it is the optimized result thanthe result 46.64>45.47<46.31 obtained by P.Kannagi,G.Uthra [2] given in the figure 2.

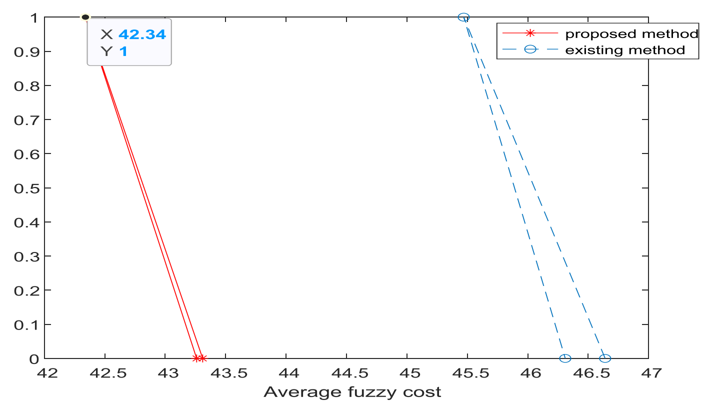



Figure 2: Comparison of results

## 5. CONCLUSIONS:

This paper's goal is to deal with uncertainty. Trapezoidal intuitionistic fuzzy numbers and triangular intuitionistic fuzzy numbers are used to represent the fuzzy parameters. In a fuzzy environment, the proposed group replacement policy is used to find the optimum solution to the problem, which is used to find the optimal replacement time without converting the fuzzy parameters into crisp ones, and a ranking method has also been developed to compare the trapezoidal intuitionistic fuzzy numbers, triangular intuitionistic fuzzy numbers, and (using Matlab). When compared to existing methods, this centroid-based ranking approach is simple and easy to implement, and we also obtained optimized results. For the proposed model, two numerical examples are also solved.

List of the notations

$\tilde{A}^{IFN}$ Intuitionistic Fuzzy Number.

$\mu_{\tilde{A}^{IFN}}$ Grade of membership function of every element $a \in \tilde{A}^{IFN}$.

$\nu_{\tilde{A}^{IFN}}$ Grade of non -membership function of every element $a \in \tilde{A}^{IFN}$.

$\alpha_{\mu_a}$ Leftspread of membership functions of $\tilde{A}^{IFN}$.

$\alpha_{\nu_a}$ Left spread of non-membership functions of $\tilde{A}^{IFN}$.

$\beta_{\mu_a}$ Right spread of membership functions of $\tilde{A}^{IFN}$.

$\beta_{\nu_a}$ Right spread of non - membership functions of $\tilde{A}^{IFN}$.

$R(\tilde{A}^{IFN})$ Ranking function of intuitionistic fuzzy number.

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