Cost-Time Trade-off of Dual Hesitant Transportation Problem with Multi Choice Parameters

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Abstract:

This paper presents a cost-time trade transportation problem with multi choice parameters in dual hesitant fuzzy environment. The problem deals with two objectives which includes cost and time. The mathematical model of cost-time trade transportation problem is formulated with dual hesitant interval supply and demand. The cost coefficients and time coefficients are considered as multi choices. The constraints have introduced to rectify the interval based dual hesitant transportation problem. The numerical problem is solved to show the efficiency of the proposed method.

Keywords: Cost-Time Trade-off , Dual Hesitant fuzzy set, Multi choice Transportation Problem
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1. Introduction :

The transportation problem plans a network of distribution of goods from different locations to different destinations with minimal costs. In 1941, Hitchcock has studied the transportation problem.The classical transportation problem includessingle objective at a time but in general there are many situations involving more objectives other that total cost. This leads to the concept of multi objective transportation problem (MOTP).

To solve the multi objective transportation problem goal programming method was introduced by Lee(1973). Zeleny (1974) solved the multi objective transportation problem by generating non dominated basic feasible solution. Diaz(1978) developed the algorithm to obtain all non dominated solutions for MOTP. Also many authors, Aneja (1979), Gupta(1983), Nomini(2017) have developed the various solution procedures to solve the MOTP. Especially minimizing cost and time simultaneously is one of the major problems in the industry. Glickman(1977), Prakash(1998) have developed a alogorithm for finding optimal cost time pairs. Many researchers, Aneja(1979),Prakash(2008) and Sonia(2018) have studied time-cost trade off transportation problem.

A transportation problem with multiple parameter is called multi-choice transportation problem. Healy(1964) have first studied the multi choice programming concept and is considered as mixed integer programming problem.Chang(2007 , 2012), Liao(2009) and Biswal (2011) have proposed various methodology for solving multi choice transportation problem in a different situations.

Due to shortage of information, insufficient data, lack of evidence, and so forth, the data for a transportation system such as availabilities, demands and conveyance capacities are not always exact but can be fuzzy or arbitrary or both. Fuzzy set was first introduced by Zadeh(1965).

Torra and Narukawahave introduced the concept of a Hesitant Fuzzy Set (HFS) in 2009. A proper definition of a HFS was given by Torra (2010) . Zhu (2012) proposed Dual-Hesitant Fuzzy Sets (DHFSs), which are an extension of HFSs that encompass fuzzy sets, intuitionistic fuzzy sets, HFSs and fuzzy multi-sets as special cases. Torra(2010) and Zhu et al introduced the basic properties of DHFSs. Thereafter, they presented the concept of DHFSs in a group forecasting problem. Also Amit Kumar et. al (2020) proposed the new ranking method for dual hesitant fuzzy element and Gurupada et. al (2019) derived the arithmetic operations on dual hesitant fuzzy numbers.

This paper presents a cost-time trade transportation problem with multi choice parameters in dual hesitant fuzzy environment. The problem deals with two objectives which includes cost and time. The mathematical model of cost-time trade transportation problem is formulated with dual hesitant interval supply and demand. The cost coefficients and time coefficients are considered as multi choices. The constraints have introduced to rectify the interval based dual hesitant transportation problem. The numerical problem is solved to show the efficiency of the proposed method.

2. Basic Definitions:

2.1 Hesitant fuzzy set :

A hesitant fuzzy set H on Y is defined in terms of a functions h(y) that returns a subset of values in the interval [0,1] once it is applied on Y i.e an element of power set of Y

h : Y → $\rho([0,1])$

Mathematically it can be stated that $H = \{ (y_i , h(y_i)) : y_i \in Y \}$ where $h(y_i)$ is a set of several values in [0,1].In general each member of $h(y_i)$ is called a hesitant fuzzy element denoted by h_i .

2.2 Example :

Consider Y = { a,b,c } . Define a hesitant fuzzy set H on Y as

H = { (a , 0.5 , 0.7, 0.78), (b, 0.8, 0.9), (c, 0.15, 0.24, 0.3,0.4)}

2.3 Dual hesitant fuzzy set:

A dual hesitant fuzzy set DH on Y is defined in terms of a functions h(y) and g(y) that returns a subset of values in the interval [0,1] once it is applied on Y

$h : Y \to \rho(Y)$and$g : Y \to \rho(Y)$

where h(y) and g(y) are mappings that takes set of values in [0,1]; they are denoted as the possible membership degree and non-membership degree of any element $y \in Y$.

Mathematically, DH$= \{ (y_i , h(y_i), g(y_i)) : y_i \in Y \}$ where $h(y_i)$ is a set of several membership values in [0,1] and $g(y_i)$is a set of several possible non-membership values in [0,1] with $0 \leq h(y_i) + g(y_i) \leq 1$.

2.4 Example :

Consider Y = { a, b, c } . Define a hesitant fuzzy set DH on Y as

DH = { (a , (0.5 , 0.7, 0.78), (0.4, 0.2,0.1)), (b, (0.8, 0.9), (0.15, 0.1)), (c,(0.15, 0.24, 0.3,0.4) , (0.8, 0.7, 0.65, 0.5))}

2.5 Arithmetic Operations of dual hesitant fuzzy elements :

Let DH1$= \{ (y_i , h_1(y_i), g_1(y_i)) : y_i \in Y \}$ and Let DH2$= \{ (y_i , h_2(y_i), g_2(y_i)) : y_i \in Y \}$ be two dual hesitant fuzzy elements. Then addition , subtraction and multiplication of the elements are defined as follows :

Addition :

DH1 ⊕ DH2 $= \cup_{\gamma h1 \in h_1 , \gamma h2 \in h_2, \delta h1 \in h_1 , \delta h2 \in h_2} \{ \{ \gamma h1 + \gamma h2 - \gamma h1 \, \gamma h2 \}, \{\delta h1 \delta h2\}\}$

Subtraction :

DH1 ⊖ DH2 $= \cup_{\gamma h1 \in h_1 , \gamma h2 \in h_2, \delta h1 \in h_1 , \delta h2 \in h_2} \{ \{ \gamma h1 \, \gamma h2 \}, \{\delta h1 + \delta h2 - \delta h1 \delta h2\}\}$

Multiplication :

DH1 $\otimes$ DH2 = $\cup_{\gamma h1 \in h_1, \gamma h2 \in h_2, \delta h1 \in h_1, \delta h2 \in h_2} \{\{\gamma h1\, \gamma h2\}, \{\delta h1 \delta h2\}\}$

2.5 Ranking function of dual hesitant fuzzy sets:

Let DH= $\{(y_i, h(y_i), g(y_i)): y_i \in Y\}$ be a dual hesitant fuzzy set where $\{y_1, y_2, \dots \dots y_n\}$ and $d = (h_d, g_d)$ be dual hesitant fuzzy element. The score function $s_d$of dual hesitant fuzzy set is defined as

$$s_d = y_i + \frac{1}{k}\sum_{i=1}^{k} h_d(y_i) - \frac{1}{k}\sum_{i=1}^{k} g_d(y_i)$$

Let d_1 and d_2 be any two dual hesitant fuzzy sets. Then using the score function the order relations are defined as follows :

1. If $s_{d_1} > s_{d_2}$, then d_1 is said to be superior to d_2 and it is denoted by $d_1 > d_2$
2. If $s_{d_1} < s_{d_2}$, then d_1 is said to be inferior to d_2 and it is denoted by $d_1 < d_2$
3. If $s_{d_1} = s_{d_2}$, then d_1 is said to be equivalent to d_2 and it is denoted by $d_1 = d_2$

3.1 Mathematical Model - 1:

Min $z_1 = \sum_{i=1}^{n}\sum_{j=1}^{m}[c_{ij}^1, c_{ij}^2, \dots c_{ij}^k]\, x_{ij}$

Min $z_2 = \max(t_{ij}(x_{ij}))$;

$(t_{ij}(x_{ij})) = [t_{ij}^1, t_{ij}^2, \dots, t_{ij}^k]$ (≥ 0) if $x_{ij} > 0$

= 0 if $x_{ij} = 0$

Subject to

$$\sum_{j=1}^{n} x_{ij} \in [\widetilde{a_{L\iota}}, \widetilde{a_{R\iota}}] \quad \forall\, i = 1,2, \dots m$$

$$\sum_{i=1}^{m} x_{ij} \in [\widetilde{b_{L\iota}}, \widetilde{b_{R\iota}}] \quad \forall\, j = 1,2, \dots n$$

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$x_{ij} \geq 0$; $i = 1,2, \dots m$, $j = 1,2, \dots n$

$\widetilde{a_{L\iota}}$and$\widetilde{a_{R\iota}}$are the lower and upper bounds for supply which are in dual hesitant fuzzy numbers and $\widetilde{b_{L\iota}}$ and $\widetilde{b_{R\iota}}$ are the lower and upper bounds for demand which are in dual hesitant fuzzy numbers. $c_{ij}^1, c_{ij}^2, \dots c_{ij}^k$are the k multi choices for cost coefficients and $t_{ij}^1, t_{ij}^2, \dots, t_{ij}^k$ are the k multi choices for time coefficients.

3.2 Mathematical Model-2:

Consider an uncapacitated directed network G (N, A) with a cost c_{ij} and y_{ij} associated with each arc (i , j) ∈ A . M_1denotes the set of m nodes corresponding to sources $s_1, s_2, \dots, s_m$ with supply $\widetilde{a_{L\iota}}$ and M_2denotes the set of n nodes corresponding to the destinations$d_1, d_2, \dots, d_n$ with demand $-\widetilde{b_{RJ}}$. $M_1^{'}$represents the set of m nodes corresponding to dummy sources $s_1^{'}, s_2^{'}, \dots, s_m^{'}$with supply $\widetilde{a_{R\iota}} - \widetilde{a_{L\iota}}$ and $M_2^{'}$ represents the set of n nodes corresponding to dummy destinations $d_1^{'}, d_2^{'}, \dots, d_n^{'}$ with demand $-(\widetilde{b_{RJ}} - \widetilde{b_{LJ}})$. S and D are correspond to super source and demand with 0 respectively. Thus the above mathematical model can be reframed as follows:

Minimize $z_1 = \sum_{(i,j)\in P} c_{ij} y_{ij}$

Subject to

$$\sum_{j:(i,j)\in P} y_{ij} - \sum_{j:(i,j)\in P} y_{ji} = b(i) \quad , \qquad \forall i \in N$$

$$y_{s_i d_j} = x_{ij} \quad , \ \forall\, i \in I \,\&\, j \in J$$

$$w_{s_i^{'} s_i} = \sum_{j=1}^{n} x_{ij} - \widetilde{a_{L_\iota}} \quad , \qquad \forall i \in I$$

$$z_{d_j' d_j} = \sum_{i=1}^{m} x_{ij} - \widetilde{b_{L_J}} \quad , \quad \forall j \in J$$
$$p_{s_i' S} = \widetilde{a_{R_l}} - \widetilde{a_{L_l}} - w_{s_i' s_i} \ , \ \forall i \in I$$
$$q_{D d_j} = \widetilde{b_{R_J}} - \widetilde{b_{L_j}} - z_{d_j' d_j} \ , \quad \forall j \in J$$
$$r_{SD} = \sum_{I=1}^{m} w_{s_i' S}$$

Where

$$N = M_1 \cup M_2 \cup M_1' \cup M_2' \cup \{S\} \cup \{D\}$$
$$M_1 = \{s_1, s_2, \dots, s_m\}$$
$$M_2 = \{d_1, d_2, \dots, d_n\}$$
$$M_1^1 = \{s_1', s_2', \dots, s_m'\}$$
$$M_2' = \{d_1', d_2', \dots, d_n'\}$$
$$P = B_1 \cup B_2 \cup B_3 \cup B_4 \cup B_5 \cup \{(S, D)\}$$

$B_1 = \{(s_i, d_j);\ \forall\, i \in I \,\&\, j \in J\}$

$$B_2 = \{(s_i', s_i);\ \forall\, i \in I\}$$
$$B_3 = \{(d_j, d_j');\ \forall\, j \in J\}$$
$$B_4 = \{(s_i', s);\ \forall\, i \in I\}$$
$$B_5 = \{(d, d_j');\ \forall\, j \in J\}$$

$c_{s_i d_j} = c_{ij}\ ;\ \forall\, i \in I, j \in J$
$= 0\ ;\ \forall\, (i,j) \in P/B_1$

$$b(s_i) = \widetilde{a_{Li}}\ , \qquad \forall i \in I$$
$$b(d_j) = -\widetilde{b_{L_J}}\ , \forall\, j \in J$$
$$b(s_i') = \widetilde{a_{R_l}} - \widetilde{a_{L_l}}\ , \forall\, i \in I$$
$$b(d_j') = -\left(\widetilde{b_{R_J}} - \widetilde{b_{L_J}}\right)\ , \forall\, j \in J$$
$$b(S) = b(D) = 0$$

4. Algorithm:

Step:1

Construct the mathematical model 2 by forming only the cost objective function and selecting the minimum cost value for multi-choice cost coefficients in model 1. Solve mathematical model 2 to optimality.

Step: 2

Generate the r-th pair is (C_r, T_r). To generate the next pair , construct multi-choice with revised cost as given below:

$$c_{ij}^k = H \quad if\ t_{ij}^k \geq T_r$$
$$= c_{ij}^k \quad otherwise$$

Where H is the highest positive number. Solve the problem by framing model 2 which is similar to model 1. If there is no feasible solution goto step 4 otherwise note C_{r+1} and T_{r+1} .

Step: 3

If $C_r < C_{r+1}$ and $T_r > T_{r+1}$ record (C_r, T_r) as the optimal cost-time pair. Goto step 2.

Step: 4

All the cost-time pairs recorded to complete the optimal pairs.

5. Numerical Example :

To illustrate the application of proposed method, consider a case where a firm XZY wishes to transport a high value in packets. These packets should be transported from 5 sources to 6 destinations such that minimum demand at each destination is arrived. Each packets may be transported through three various airlines $X_1, X_2\ and\ X_3$. The cost and time quotes per unit corresponding to these three different airlines are given in the table 1 and 2 as multi-choices. Aim of this problem is to obtain the optimal cost-time pair.

Table: 1 Cost Coefficients :

	L1	L2	L3	L4	L5	L6	Supply
N1	(18,29,10)	(29,20,15)	(17,25,16)	(10,27,13)	(20,28,19)	(24,14,13)	≥ (11 ; 0.7,0.65; 0.2, 0.3) and ≤ (12;0.8, 0.83, 0.85; 0.11, 0.12, 0.1)
N2	(15,25,m)	(27,23,m)	(19,23,19)	(26,17,18)	(22,18,15)	(27,12,26)	≥ (23; 0.7,0.75,0.8; 0.2,0.23,0.25) and ≤ (37;0.5,0.55,0.6; 0.2, 0.25, 0.28)
N3	(19,27,27)	(22,16,27)	(22,23,14)	(19,13,20)	(29,10,26)	(24,26,11)	≥ (26; 0.75, 0.73, 0.72, 0.7; 0.21, 0.23, 0.25) and ≤ (33 ; 0.6, 0.63; 0.3,0.35,0.38,0.39)
N4	(m,20,23)	(27,20,24)	(27,16,14)	(14,17,19)	(17,15,19)	(13,20,22)	≥(7; 0.8,0.82,0.85; 0.09,0.08,0.1) and ≤ (8; 0.9,0.8,0.85; 0.08,0.07,0.09)
N5	(19,14,24)	(18,13,24)	(28,26,17)	(13,28,m)	(20,18,29)	(14,18,27)	≥ (33;0.7,0.75,0.78; 0.15,0.18,0.2) and ≤ (76; 0.6,0.5,0.3 ; 0.2,0.25)
Demand	≥ (13; 0.7,0.75; 0.2,0.25) and ≤(23; 0.8,0.82; 0.1,0.17)	≥(31; 0.6,0.63,0.62; 0.2,0.25,0.28) and ≤ (39; 0.7,0.73; 0.25,0.24)	≥ (8; 0.65;0.68; 0.3,0.28, 0.27) and ≤ (11; 0.5,0.52, 0.55; 0.3,0.4)	≥(21; 0.8,0.81 ; 0.15,0.18) and ≤ (22; 0.6,0.5; 0.3,0.2)	≥(5; 0.7,0.6,0.5; 0.3,0.2) and ≤ (7; 0.72,0.73; 0.2,0.25)	≥(22 ; 0.83,0.81; 0.15, 0.13, 0.12) and ≤ (64; 0.8, 0.75; 0.15,0.2)	

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Table: 2 Time Coefficients

	L1	L2	L3	L4	L5	L6
N1	(974,1039,818)	(1105,1116,1192)	(1023,948,1109)	(894,898,1198)	(1089,1153,1051)	(1089,1153,1051)
N2	(934,946,m)	(840,976,m)	(1170,866,1097)	(1131,1089,862)	(955,997,1173)	(1146,1179,1103)
N3	(924,1049,973)	(857,1070,1064)	(1052,1157,906)	(1009,1018,1166)	(1124,816,901)	(1130,957,1076)
N4	(m,908,1142)	(1128,800,973)	(904,1027,1198)	(1196,1157,1116)	(1035,867,815)	(850,1057,1116)
N5	(936,924,1082)	(1124,1113,1164)	(890,911,1047)	(1081,1010,m)	(915,1069,1169)	(1114,896,1164)

Select the minimum cost and its corresponding time from table 1 and 2 then the multi-choice problem becomes single choice as given below. Also use the ranking function dual hesitant fuzzy number can be rewritten as follows:

Table:3 Selected cost and time coefficients

	L1	L2	L3	L4	L5	L6	Supply
N1	10 ; 818	15; 1192	16 ; 1109	10 ; 894	19 ; 1051	13 ; 1074	≥ 10.575 and ≤11.283
N2	15 ; 934	23 ; 976	19 ; 1097	17 ; 1089	15 ; 1173	12 ; 1179	≥22.18 and ≤36.69
N3	19 ; 924	16 ; 1070	14 ; 906	13 ; 1018	10 ; 1173	11 ; 1076	≥25.49 and ≤32.74
N4	20 ; 908	20 ; 800	14 ; 1198	14 ; 1196	15 ; 867	13 ; 850	≥6.26 and ≤7.28
N5	14 ; 924	13 ; 1113	17 ; 1047	13 ; 1081	18 ; 1069	14 ; 1114	≥ 32.44 and ≤75.725
Demand	≥12.49 and ≤ 22.325	≥30.62 and ≤ 38.53	≥7.615 and ≤ 10.88	≥20.32 and ≤21.7	≥4.65 and ≤ 6.5	≥21.31 and ≤63.4	

Using model 2 the above problem can be formulated as given below:

Min z_1 = 10y11 + 15y12 + 17y13 + 13y14 + 19y15 + 13y16+15y21 + 23y22 + 19y23 + 17y24 + 15y25 + 12y26 + 19y31 + 16y32 + 14y33 + 13y34 + 10y35 + 11y36+ 20y41 + 20y42 + 14y43 + 14y44 + 15y45 + 13y46+ 14y51 + 13y52 + 17y53 + 13y54 + 18y55 + 14y56

w11= y11+y12+y13+y14+y15+y16-10.575
w22= y21+y22+y23+y24+y25+y26-22.48
w33= y31+y32+y33+y34+y35+y36-25.49
w44= y41+y42+y43+y44+y45+y46-6.26
w55= y51+y52+y53+y54+y55+y56-32.44
z11= y11+y21+y31+y41+y51-12.49
z22= y12+y22+y32+y42+y52-30.62
z33= y13+y23+y33+y43+y53-7.615
z44= y14+y24+y34+y44+y54-20.32
z55= y15+y25+y35+y45+y55-4.65
z66= y16+y26+y36+y46+y56 -21.31
p11 = 11.283-10.575-w11
p12 = 36.69-22.48-w22
p13 = 32.74-25.49-w33
p14 = 7.28-6.29-w44
p15 = 75.725-32.44-w55
q11= 22.325-12.49-z11
q12= 38.53-30.62-z22
q13= 10.88-7.615-z33
q14= 21.7-20.32-z44
q15= 6.5-4.65-z55
q16= 63.4-21.31-z66
r11= p11+p22+p33+p44+p55

with the help of LINGO software above model can be solved and allocated cells are shown below:

Table: 4 Iteration -1

	L1	L2	L3	L4	L5	L6	Supply
N1	**10 ; 818**	15; 1192	16 ; 1109	10 ; 894	19 ; 1051	13 ; 1074	≥ 10.575 and ≤ 11.283
N2	**15 ; 934**	23 ; 976	19 ; 1097	**17 ; 1089**	15 ; 1173	**12 ; 1179**	≥ 22.18 and ≤ 36.69
N3	19 ; 924	16 ; 1070	**14 ; 906**	**13 ; 1018**	**10 ; 1173**	**11 ; 1076**	≥ 25.49 and ≤ 32.74
N4	20 ; 908	20 ; 800	**14 ; 1198**	14 ; 1196	15 ; 867	13 ; 850	≥ 6.26 and ≤ 7.28
N5	14 ; 924	**13 ; 1113**	17 ; 1047	**13 ; 1081**	18 ; 1069	14 ; 1114	≥ 32.44 and ≤ 75.725
Demand	≥ 12.49 and ≤ 22.325	≥ 30.62 and ≤ 38.53	≥ 7.615 and ≤ 10.88	≥ 20.32 and ≤ 21.7	≥ 4.65 and ≤ 6.5	≥ 21.31 and ≤63.4	

Optimal cost = Rs. 1207.18
Here H = max { 818, 934 , 109, 1179, 906, 1018, 1173, 1076, 1198, 1113, 1081} = 1198
Hence the first optimal pair is (1210.18, 1198).
Use step 2, replace the cost in the cell(4,3) as H (Table 5) and solve with the help of LINGO.

Table: 5 Iteration- 2

	L1	L2	L3	L4	L5	L6	Supply
N1	**10 ; 818**	15; 1192	16 ; 1109	10 ; 894	19 ; 1051	13 ; 1074	≥ 10.575 and ≤ 11.283
N2	**15 ; 934**	23 ; 976	19 ; 1097	**17 ; 1089**	15 ; 1173	**12 ; 1179**	≥ 22.18 and ≤ 36.69
N3	19 ; 924	16 ; 1070	**14 ; 906**	**13 ; 1018**	**10 ; 1173**	11 ; 1076	≥ 25.49 and ≤ 32.74
N4	20 ; 908	20 ; 800	H	**14 ; 1196**	15 ; 867	13 ; 850	≥ 6.26 and ≤ 7.28
N5	**14 ; 924**	**13 ; 1113**	17 ; 1047	**13 ; 1081**	18 ; 1069	14 ; 1114	≥ 32.44 and ≤ 75.725
Demand	≥ 12.49 and ≤ 22.325	≥ 30.62 and ≤ 38.53	≥ 7.615 and ≤ 10.88	≥ 20.32 and ≤ 21.7	≥ 4.65 and ≤ 6.5	≥ 21.31 and ≤63.4	

Hence second optimal pair is (1213.44 , 1196).
Similarly , list of optimal pairs are (1234.765,1179), (1283.46, 1173), (1297.46, 1113), (1493.79, 1081) and (1653, 1076).

Hence the optimal cost is Rs. 1210.18 and optimal time is 1076.

6. Conclusion :

This paper presents a cost-time trade transportation problem with multi choice parameters in dual hesitant fuzzy environment. The mathematical model of cost-time trade transportation problem is formulated with interval supply and demand. The problem deals with two objectives which includes cost and time. The constraints have introduced to rectify the interval based dual hesitant transportation problem. Dual hesitant fuzzy environment is very useful to tackle the uncertainty of the real life problem, more over it helps to reduce the cost than other fuzzy situation. The real life problem is solved using with help of proposed method.

7. References

1. AbouzarSheikhi (2018), A new Methosd for solving Bi-Objecive Fractional Transportation Problem, ISSN : 0974-2328
2. Amit Kumar et. al (2020) , Mehar approach for solving dual-hesitant fuzzy transportation problem with restrictions , Indian Academy Sciences, Vol. 45(77), 1-9.
3. Aneja YP and Nair KPK , (1979) , Bi-criterial transportation problem, Manag, Sci., Vol. 25, 73 – 78
4. Ather Aziz Raina , (2018) Fractional Transportation Problem with non-linear discount cost, Sri Lankan Journal of Applied Statistics
5. Borza , Rambely , Saraj , " A new method for solving fractional programming problems with interval coeffients in the objective function, Applied mathematics sciences ,6, 3445-3452(2012
6. Charnes A and Cooper W, (1962) Program with linear fractional functions, Naval Research Logistics Quarterly.
7. Chakraborty , Gupta(2013), Fuzzy mathematics programming for multi-objective fraction programming problem, Fuzzy Sets and Systems, Vol.123 (3), 559-567
8. Diaz JA (1978), Solving multiobjective transportation problems, Ekonomickymatematickyobzor, Vol. 14, 267 – 274
9. Diaz JA (1979), Finding a complete description of all efficient solution to a multiobjective transportation problem ,Ekonomickymatematickyobzor, Vol. 15, 62 -73 .
10. Glickman TS and Berger PD(1977) , Technical notecost/completion-date tradeoffs in the transportation, Operations Research, Vol.25(1),163-168.
11. Gupta B and Gupta R (1983), Multi-criteria simplex method for a linearmultiple objective transportation problem, Indian J. Pure Appl. Math, Vol. 4, No. 2 , 222 – 232 .
12. Gurupadaet. al (2019), A new approach for solving dual- hesitant fuzzy transportation problem with restrictions, Indian Academy of Sciences, Vol. 44(75), 1-11.
13. Gupta N , Ali, I and Bari , A (2013) , A compromise solution for multi-objective chance constraint capacitated transportation problem, Probstat Forum, Vol. 6, No. 4 , 60 – 67.
14. Hitchcock , FL (1941), The distribution of a product from several sources to numerous localities, J.Math. Phy., Vol. 20 , 224 – 230 .
15. LakhveerKaur et al (2018), A new approach to solve multi objective transportation problem, Applications and applied mathematics, Vol. 13 (1), 150 - 159 .
16. Lee , SM and Moore LJ (1973), Optimizing transportation problems with multiple objective, AIEE Transactions, Vol. % , 333-338.
17. NomaniMA , Ali L and Ahmed A (2017), A new approach for solving multi-objective transportation problems, Internation journal of manament science and Engineering Management, Vol. 12 , 165 – 173 .
18. Prakash S, Agarwal AK, &Shah (1998), Nondominated solutions of cost-time trade-off trandportation and assignment problems, Opsearch , Vol.25(2),126-131.
19. Prakash S, Kumar P, Prasad B and Gupta A(2008), Pareto optimal solutions of cost-time trade-off transportation problem, European Journal of Operational Research, Vol.188(1),85-100.
20. Pop B, Stancu-Minasian(2008), A method of solving fully fuzzied linear fractional programming problems, Journal of Applied Mathematical Computing , Vol 28, 227-242.
21. Sheema et.al (2017), Multiobjective fractional transportation problem in fuzzy environment, International Journal of Mathematical Archive, 8(12), 2017, 197-209
22. Sonia Singh and Shalabh Singh, (2018), Bi-criteria transportation problem with

multiple parameters, Annals of Operation research,
23. Torra V and Narukawa Y 2009 On hesitant fuzzy sets and decision. In: Proceedings of the IEEE International Conference on Fuzzy Systems. https://doi.org/10.1109/FUZZY. 2009.5276884, pp. 1378–1382
24. Torra V 2010 Hesitant fuzzy sets. International Journal of Intelligent Systems 25(6): 529–539
25. Viwas Deep Joshi(2020), On a Paradox in Multi-Objective Linear and Fractional Transportation Problem, Science & Technology Asia , Vol. 25, 157-165.
26. Zeleny M, (1974), Linear multiobjective programming , Springer – Verlag, Berlin.
27. Zhu B, Xu Z and Xia M 2012 Dual hesitant fuzzy sets. Journal of Applied Mathematics.