Analyzation of Multiserver Queuing Models with Reverse Balking and Impatient Customers Under Imprecise environment

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Abstract.

Multiserver fuzzy queuing system with reverse balking and impatient customers is being investigated in this article. Customer impatience has a damaging influence on businesses since it causes them to lose potential consumers, which has a negative impact on the entire company. The concept of reverse balking is novel in stochastic queuing models. Recursively, the model's steady-state solution is obtained. To convert the times of arrival and service into a crisp value using wingspans ranking method. The significant measures such as expected waiting time, average rate of reneging and average rate of reverse balking are obtained. This method is exemplified using numerical examples.

Keywords. Customers' Impatience, Retention Of Customers, Wingspans Fuzzy Ranking, Multi-server Fuzzy Queuing Model, Fuzzy Number, Triangular Fuzzy Number, Reverse Balking, Reneging.

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I. LITERATURE REVIEW AND INTRODUCTION

Analyze how consumers arrive for service, how long they wait for service when it's not available right away, and then exiting the system after they have been served are both examples of queuing theory. Since the arrival process is stochastic in most queuing situations, understanding the probability distribution representing the time duration between consecutive consumer arrivals is critical. Customers must be able to arrive in batches (batch or bulk arrivals), and if so, the likelihood function representing batch size must be available. It's also important to understand how a customer reacts when they first join the system. A customer can choose to wait regardless of how long the line grows, or the customer can decide not to enter the system if the line becomes too long. If a customer declines to enter the queue when they arrive, they are said to have balked.

A customer can enter the queue at any time, but after a period of time, lose patience and leave. The customer is said to have reneged in this situation. Customers can move from one waiting line to another if there are two or more parallel lines. This is known as jockeying for position. These three scenarios are all examples of customers who are impatiently waiting in lines. In real-life situations, queuing theory is crucial. Nowadays, we have a lot of queuing issues in places like hospitals, medical stores, ration shops, reservation centers, ATM machines, and so on....;

Basic preliminaries are generally discussed in Bose. S ([5]), Cooper. R ([8]), Janos. S ([15]), as well as queuing models, are critical for our study. We use fuzzy logic and applications are generally concentrated in Klir.G.J. andYuvan.B. [16], Kumar. R. and Sharma S.K. [20] in real-life scenarios the majority of the time. Finding impatience to be a challenge to business, companies employ a variety of tactics to keep a reneging customer, and they usually succeed. Many authors have suggested different methods for determining the output of fuzzy queues. In order to evaluate the performance measures of fuzzy queues, our proposed ranking method will transform from a fuzzy to a crisp environment. Westman.L and Wang.Z [25] used their left and right wingspans to rank fuzzy numbers. Some writers, in particular, Ramesh. R and Kumaraghuru S. [22], Shankar. N.R, Sarathi.B.P. &Babu.S.S.[23], and Wang. Y.J. and Lee. H.S. [24] has been used centroid-based ranking techniques. In this paper, sensitive business with customer impatience is considered as a fuzzy queuing process, When an M/M/C/N configuration with finite system capacity is handling arguments, and the queue discipline is FCFS. Reverse balking and reneging are used in this model.

II. MODEL PREDICTIONS

1. Individuals arrive at a fuzzy queuing mechanism one by one, according to a Poisson process with mean arrival rate $\tilde{\lambda}$.
2. The regulation claims are stored in parallel on a multi-server system. The service hours are equal and independent of one another uniformly, and with a fuzzy parameter $\tilde{\mu}$ that is exponentially distributed, such $\tilde{\mu} = n\tilde{\mu}$ for $n<c$, $\tilde{\mu} = c\tilde{\mu}$ for $n \geq C$.
3. The mechanism has a limited capacity, say *N*.
4. Policy demands are evaluated in the order in which they are provided; this is known as the First-come, First-served queue discipline.

(a) Consumers balk with probability when the system is empty and can purchase with probability $p' = (1-q')$ $p^1 = \left(1-q^1\right)$.

(b) Consumers balk with a probability $1-\frac{n}{N-1}$ and enter the scheme with a probability $\frac{n}{N-1}$, where there is at least one customer in the system; this kind of balking is known as reverse balking.

5. The reneging time (T) is distributed independently, identically, and exponentially for parameter $\tilde{\xi}$.

A) PRELIMINARIES

1) Definitions

1.1. Fuzzy set

$\tilde{A}$ is a fuzzy set defined on X and written as a collection of ordered pairs if X is a universe of discourse and x is a particular element of X.

$$A = \left\{\left(x, \mu_{\tilde{A}}(x)\right), x \in X\right\}.$$

1.2 Fuzzy number

- Set $\tilde{A}$: R $\rightarrow$ [0,1]
- To qualify as fuzzy number, a fuzzy set $\tilde{A}$ on R must have following properties

1. Set $\tilde{A}$ must be Normal fuzzy set
2. α-cut of $\tilde{A}$ must be closed interval for every α in (0,1];
3. Support and strong α-cut of $\tilde{A}$ must be bounded.

4. Every fuzzy number is a convex fuzzy set; the inverse is not necessarily true.

1.3 Triangular fuzzy number

A fuzzy number $\tilde{A}=(a,b,c)$ is called triangular fuzzy number if its membership function is given by:

$$\mu_{\tilde{A}}(x)=\begin{cases}0 & , x<a \; ; \\ \frac{x-a}{b-a}, & a\le x\le b; \\ \frac{c-x}{c-b}, & b\le x\le c; \\ 0 & , x>c\end{cases}$$

III. STOCHASTIC MODEL

A Differential equation of the model is given by:

$$\frac{dPo(t)}{dt}=-\tilde{\lambda}\, p' P_0(t)+\tilde{\mu}\, P_1(t); n=0 \qquad \rightarrow (1)$$

$$\frac{d\, P_1(t)}{dt}=\tilde{\lambda}\, p' P_0(t)-\left\{\left(\frac{1}{N-1}\right)\tilde{\lambda}+\tilde{\mu}\right\}P_1(t)+(2\tilde{\mu})P_2(t); n=1 \qquad \rightarrow (2)$$

$$\frac{dP_n(t)}{dt}=\left(\frac{n-1}{N-1}\right)\tilde{\lambda}\, P_{n-1}(t)-\left\{\left(\frac{n}{N-1}\right)\tilde{\lambda}+n\tilde{\mu}\right\}P_n(t)+\{(n+1)\tilde{\mu}\}P_{n+1}(t); 2\le n<c \qquad \rightarrow (3)$$

$$\frac{dP_n(t)}{dt}=\left(\frac{n-1}{N-1}\right)\tilde{\lambda}P_{n-1}(t)-\left\{\left(\frac{n}{N-1}\right)\tilde{\lambda}+c\tilde{\mu}+(n-c)\tilde{\xi}\right\}P_n(t)$$
$$+\left[c\tilde{\mu}+\{(n+1)-c\}\tilde{\xi}\right]P_{n+1}(t); n\ge c \qquad \rightarrow (4)$$

$$\frac{dP_N(t)}{dt}=\tilde{\lambda}P_{N-1}(t)-\left[c\tilde{\mu}+(N-c)\tilde{\xi}\right]P_N(t); n=N \qquad \rightarrow (5)$$

IV. STEADY – STATE SOLUTION

In steady state, $\lim_{t\to\infty}P_n(t)=P_n, \lim_{t\to\infty}P_n'(t)=0$. Therefore the equations (1) to (5) becomes,

$$0=-\tilde{\lambda}p' P_0+\tilde{\mu}P_1 \qquad ; n=0 \qquad \rightarrow (6)$$

$$0=\tilde{\lambda}p' P_0-\left\{\left(\frac{1}{N-1}\right)\tilde{\lambda}+\tilde{\mu}\right\}P_1+(2\tilde{\mu})P_2; n=1 \qquad \rightarrow (7)$$

$$0=\left(\frac{n-1}{N-1}\right)\tilde{\lambda}P_{n-1}-\left\{\left(\frac{n}{N-1}\right)\tilde{\lambda}+n\tilde{\mu}\right\}P_n+\{(n+1)\tilde{\mu}\}.P_{n+1}; 2\le n<c \qquad \rightarrow (8)$$

$$0=\left(\frac{n-1}{N-1}\right)\tilde{\lambda}P_{n-1}-\left\{\left(\frac{n}{N-1}\right)\tilde{\lambda}+c\tilde{\mu}+(n-c)\tilde{\xi}\right\}P_n+\left[c\tilde{\mu}+\{(n+1)-c\}\tilde{\xi}\right]P_{n+1}; n\ge c \rightarrow (9)$$

$$\frac{dP_N(t)}{dt}=\tilde{\lambda}P_{N-1}(t)-\left[c\tilde{\mu}+(N-C)\tilde{\xi}\right]P_N(t); n=N \qquad \rightarrow (10)$$

The model's steady-state solution is obtained by solving (6) - (10) iteratively. The probability of *n* customers in the method can be calculated as follows:

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$$P_n = \begin{cases} \left[\dfrac{(n-1)!}{(N-1)^{n-1}} \prod_{r=1}^{n} \dfrac{\tilde{\lambda}}{r\tilde{\mu}} \right] p^{'} P_0, n < c \\ \left[\dfrac{(n-1)!}{(N-1)^{n-1}} \prod_{S=c}^{n} \dfrac{\tilde{\lambda}}{c\tilde{\mu} + (S-c)\tilde{\xi}} \prod_{r=1}^{c-1} \dfrac{\tilde{\lambda}}{c\tilde{\mu}} \right] p^{'} P_0, n \geq c \\ \left[\dfrac{(N-2)!}{(N-1)^{N-1}} \prod_{S=c}^{n} \dfrac{\tilde{\lambda}}{c\tilde{\mu} + (S-c)\tilde{\xi}} \prod_{r=1}^{c-1} \dfrac{\tilde{\lambda}}{r\tilde{\mu}} \right] p^{'} P_0, n = N \end{cases}$$

Using the normalization condition $\sum_{n=1}^{N} P_n = 1,$ we get

$$P_0 + \sum_{n=1}^{c-1} P_n + \sum_{n=c}^{N-1} P_n + P_N = 1$$

$$P_0 = \left\{ 1 + \left[\frac{(n-1)!}{(N-1)^{n-1}} \prod_{r=1}^{n} \frac{\tilde{\lambda}}{r\tilde{\mu}} \right] p^{'} + \left[\frac{(n-1)!}{(N-1)^{n-1}} \prod_{S=c}^{n} \frac{\tilde{\lambda}}{c\tilde{\mu} + (s-c)\tilde{\xi}} \prod_{r=1}^{c-1} \frac{\tilde{\lambda}}{r\tilde{\mu}} \right] p^{'} + \left[\frac{(N-2)!}{(N-1)^{N-1}} \prod_{s=c}^{n} \frac{\tilde{\lambda}}{c\tilde{\mu} + (s-c)\tilde{\xi}} . \prod_{r=1}^{c-1} \frac{\tilde{\lambda}}{r\tilde{\mu}} \right] p^{'} \right\}^{-1}$$

V. MEASURES OF PERFORMANCE

5.1 Expected system size

$$L_s = \sum_{n=1}^{N} nP_n.$$

$$L_s = \sum_{n=1}^{c-1} np_n + \sum_{n=c}^{N-1} nP_n + NP_N$$

$$L_s = \sum_{n=1}^{c-1} n \left[\frac{(n-1)!}{(N-1)^{n-1}} \prod_{r=1}^{n} \frac{\tilde{\lambda}}{r\tilde{\mu}} \right] p^{'} P_0 + \sum_{n=c}^{N-1} n \left[\frac{(n-1)!}{(N-1)^{n-1}} \prod_{s=c}^{n} \frac{\tilde{\lambda}}{c\tilde{\mu} + (s-c)\tilde{\xi}} \prod_{r=1}^{c-1} \frac{\tilde{\lambda}}{r\tilde{\mu}} \right] p^{'} P_0$$

$$+ N \left[\frac{(N-2)!}{(N-1)^{N-1}} \prod_{S=c}^{n} \frac{\tilde{\lambda}}{c\tilde{\mu} + (s-c)\tilde{\xi}} \prod_{r=1}^{c-1} \frac{\tilde{\lambda}}{r\tilde{\mu}} \right] p^{'} P_0.$$

5.2. Average rate of reneging

$$R_r = \sum_{n=c}^{N} (n-c)\tilde{\xi} \, P_n$$

$$R_r = \sum_{n=c}^{N-1} (n-c) \left[\frac{(n-1)!}{(N-1)^{n-1}} \prod_{s=c}^{n} \frac{\tilde{\lambda}}{c\tilde{\mu} + (s-c)\tilde{\xi}} \prod_{r=1}^{c-1} \frac{\tilde{\lambda}}{r\tilde{\mu}} \right] \tilde{\xi} p^{'} P_0$$

$$+ (N-c)\tilde{\xi} \left[\frac{(N-2)!}{(N-1)^{N-1}} \prod_{s=c}^{n} \frac{\tilde{\lambda}}{c\tilde{\mu} + (s-c)\tilde{\xi}} \prod_{r=1}^{c-1} \frac{\tilde{\lambda}}{r\tilde{\mu}} \right] \tilde{\xi} p^{'} P_0$$

5.3. Average rate of reverse balking

$$R_b^{'} = q^{'} \tilde{\lambda} P_0 + \sum_{n=1}^{N-1} \left(1 - \frac{n}{N-1} \right) \tilde{\lambda} P_n$$

$$R_b^{'} = q^{'} \tilde{\lambda} \, P_0 + \sum_{n=1}^{c-1} \left(1 - \frac{n}{N-1} \right) \left[\frac{(n-1)!}{(N-1)^{n-1}} \prod_{r=1}^{n} \frac{\tilde{\lambda}}{r\tilde{\mu}} \right] p^{'} P_0$$

$$+\sum_{n=c}^{N-1}\left(1-\frac{n}{N-1}\right)\tilde{\lambda}\left[\frac{(n-1)!}{(N-1)^{n-1}}\prod_{s=c}^{n}\frac{\tilde{\lambda}}{c\tilde{\mu}+(s-c)\tilde{\xi}}\prod_{r=1}^{c-1}\frac{\tilde{\lambda}}{r\tilde{\mu}}\right].p^{'}P_0$$

5.4. Wingspans fuzzy ranking method – Algorithm

Let $\tilde{A}$ be a fuzzy number, $\psi_{\tilde{A}}$ be its membership function, and let a_0 be one of its core point of $\tilde{A}$. The Wingspans core is then referred to as $w_{\tilde{A}} = a_0 - \frac{1}{2}\int_{-\infty}^{a_0}\psi_{\tilde{A}}(x)dx + \frac{1}{2}\int_{a_0}^{\infty}\psi_{\tilde{A}}(x)dx$ is called the wingspans center of $\tilde{A}$.

It is self-evident that the wingspans centre is the symmetric centre of every fuzzy number.

For triangular fuzzy number,

$\tilde{A}=[a_l,a_0,a_r]$, its wingspans center and also our proposed ranking function are:

$$R(\tilde{A})=\frac{1}{2}a_0+\frac{1}{4}(a_l+a_r)$$

VI. NUMERICAL EXAMPLE

Here we take the probabilities of the reneged (*q′*), the complement of *q′* is (*p′*) denoted as *p′* = 1, 0.9, 0.8...0.1, 0, *q′* = 0, 0.1, 0.2... 0.9, 1 and *N* = 15. At this stage, we are working out the expected system size, average reneging rate, and average reverse balking rate.

A: For Triangular fuzzy number

Consider the arrival rate $\tilde{\lambda}=[10,11,12]$ and the service rate $\tilde{\mu}=[13,14,15]$, and time distribution parameter $\tilde{\xi}=[0.1,0.2,0.3]$ per hour respectively.

Now the membership function of the triangular fuzzy number [10, 11, 12] is:

$$\psi_{\tilde{\lambda}}(x)=\begin{cases}\frac{x-10}{11-10}, 10\le x\le 11\\ 10 \quad , x=10\\ \frac{x-12}{11-12}, 11\le x\le 12\\ 0 \quad , otherwise\end{cases}$$

In the same way, we will continue with the remaining triangular fuzzy rates. The triangular fuzzy numbers are now ranked using the wingspans fuzzy ranking system.

$R(\tilde{\lambda})=R(10,11,12)$

$=\frac{1}{2}(11)+\frac{1}{4}(10+12)$

$R(\tilde{\lambda})=11$, Similarly

$R(\tilde{\mu})=14$, $R(\tilde{\xi})=0.2$

We tabulate the following empirical results for various performance metrics based on our equations. The variance in output measurements with respect to q′ is calculated here.

VII.NUMERICAL ILLUSTRATIONS

Let us consider $\tilde{\lambda}=(10,11,12),\tilde{\mu}=(13,14,15),\tilde{\xi}=(0.1,0.2,0.3),c=3,N=15.$

7.1. Wingspans ranking method

$\tilde{A}=[a_l,a_0,a_r]$, For triangular fuzzy number: $R(\tilde{A})=\frac{1}{2}a_0+\frac{1}{2}[a_l+a_r]$. Here, for triangular fuzzy numbers $\tilde{\lambda}=(10,11,12),\tilde{\mu}=(13,14,15)$ .We obtain,

$$R(\tilde{\lambda}) = R(10,11,12) = \frac{1}{2}(11) + \frac{1}{4}(10+12)$$

$$R(\tilde{\lambda}) = 11$$

$$R(\tilde{\mu}) = R(13,14,15) = \frac{1}{2}(14) + \frac{1}{4}(13+15)$$

$$R(\tilde{\mu}) = 14$$

$$R(\tilde{\xi}) = R(0.1,0.2,0.3) = \frac{1}{2}(0.2) + \frac{1}{4}(0.4) = 0.2$$

TABLE 1.

Performance measures vs. q′

S.No.	q′	p′	L_S	R_r	R_b'
1.	0	1	0.82983	0.00014	0.81970
2.	0.1	0.9	0.74685	0.00013	1.83773
3.	0.2	0.8	0.66386	0.00011	2.85576
4.	0.3	0.7	0.58088	0.000098	3.87379
5.	0.4	0.6	0.49790	0.000084	4.89182
6.	0.5	0.5	0.41492	0.000070	5.90985
7.	0.6	0.4	0.33193	0.000056	6.92788
8.	0.7	0.3	0.24895	0.000042	7.94591
9.	0.8	0.2	0.16597	0.000028	8.96394
10.	0.9	0.1	0.08298	0.000014	9.98197
11	1	0	0.00000	0	1

RESULT

The above finding shows that as the chance of keeping a reneging customer increase, the average rate of reverse balking q′ increases as well. When q′ = 0, the average reneging rates fall, and R_r = 0.00014. Furthermore, when q′ = 1, R_r = 0, all reneging customers are retained.

CONCLUSION

This paper develops an M/M/C/N fuzzy Markovian queuing system of reverse balking and consumer reneging. The steady-state solution of the model is determined. As required, performance metrics are gathered. Numerical findings are possible to obtain. In this article, we look at how to find performance measures for multi-server fuzzy queuing models of reverse balking and impatient consumers using the fuzzy Wingspans ranking methodology. This method is not only produces crisp results, but it also does so with more precision than most other methods.

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