Exploration of Single Server Fuzzy Queuing Model With Erlang Service

W. Ritha and N. Rajeswari
Department of Mathematics,
HolyCrossCollege (Autonomous),
Affiliated to BharathidasanUniversity,
Tiruchirappalli – 620 002.
Tamil Nadu, India

Abstract

Fuzzy Queues are more absolute than the naturally occurred crisp queues in many real life situations**.** In this Research article, the L-R method is applied for listing execution of performance measures of single server fuzzy queuing system with Erlang service. L-R method has a superiority of being simple, flexible collated to the notable technique called alpha-cut method.A mathematical model is outlined to legitimize the legitimacy of this methodology. The numerical example is illustrated successfully for triangular fuzzy number.

Keywords: Fuzzy queuing model, Fuzzy number, α-cut method, L-R method, Non-Linear programming method, Triangular fuzzy number, Erlang service.
DOINumber:10.14704/nq.2022.20.8.NQ22969 **NeuroQuantology 2022;20(8):9504-9514**

1. Introduction

Queuing hypothesis assumes a significant job in our everyday life. It is absurd to precisely decide the appearance and takeoff of clients when the number and kinds of offices just as the basic of the clients are not known. Queuing hypothesis is the numerical perseverance of holding up lines or queues. Queue lengths and waiting time can be meant by building the queuing model. The framework where single queues are partnered by steering the organization.

The fuzzy queues are first proposed by R.J.Lie and E.S.Lee in 1989. In poisson appearance queuing framework is a genuinely sensible allocation where the arrival rate and service rate are truly more possibilistic than probabilistic. In any case, in numerous down to earth circumstances the parameters λ arrival rate and μ (service rate) are as often as possible fuzzy and can't be communicated in careful terms.

In fuzzy logic liteature, fuzzy queues are generally concentrated in R.J.Li and E.S.Lee([1],[2]),W. Ritha, S.B. Menon([4],[6],[13]),D.S. Negi, E.S. Lee[5],J.b. Mukeba Kanyinda([7],[9]),S.P.Chen([14],[15])and so on, The vast majority of the papers are signified to discover framework execution estimates utilizing alpha-cuts method. In this exploration article, the L-R fuzzy number, arithmetic of L-R fuzzy number and three-sided fuzzy number ideas are used for analysis of fuzzy queuing models.

In this paper, L-R fuzzy number, arithmetic of L-R fuzzy numbers ideas are used for discovering execution measures for single worker Erlang queuing model ($FM/FE_K/1: \infty/FIFO$).Here we compute the number of customers and the waiting time in a fuzzy queue by another method called L-R technique, basically dependent on L-R fuzzy number-arithmetic.

The Research article is coordinated as follows: second segment give some basic definitions of this research work, Third segment details a mathematical model which figures number of customers and waiting time in the queue and furthermore in the framework utilizing L-R technique, fourth segment briefly introduces the solution procedure- momentarily presents the arrangement strategy, Fifth segment outlined the numerical model in multiple times independently, utilizing progressively alpha-cuts technique and L-R method, in plan to show L-R strategy points of interest 6th and last area finishes up this new methodology.

2. Preliminaries

Definition 2.1

(i) A **fuzzy set** $\tilde{A}$ in a universe of discourse X is defined as an ordered pair $\tilde{A}=\{(x,\mu_{\tilde{A}}(x)):x\in X\}$.Here $\mu_{\tilde{A}}:X\rightarrow[0,1]$ is a mapping called the membership value of $x\in X$ in a fuzzy set $\tilde{A}$.

(ii) Let X be a classical set or a universe. A **fuzzy subset** $\tilde{A}$ (or a fuzzy set $\tilde{A}$), in X is defined by the function $\eta_{\tilde{A}}$ called membership function of $\tilde{A}$, from E to the real unit interval [0,1]

$\eta_{\tilde{A}}$ (a) is called the grade or the membership degree of a, $\forall\, a\in\tilde{A}$

Definition 2.2

Let $\tilde{A}$ be a subset in the universe X. The **support** supp $(\tilde{A})$ and the **height**, hgt $(\tilde{A})$ of $\tilde{A}$ are the crisp sets defined respectively as follows:

$$\text{Supp}(\tilde{A})=\{x\in E/\eta_{\tilde{A}}(x)>0\}$$

and $\text{hgt}(\tilde{A})=\max\{\eta_{\tilde{A}}(x)/x\in X\}$

Definition 2.3

Let $\tilde{A}$ be a subset in the universe X. The **α-cut**of $\tilde{A}$ noted $\tilde{A}_\alpha$ is a classical set defined as follows:

$$\tilde{A}_\alpha=\{x\in X/\eta_{\tilde{A}}(x)\geq\alpha\}$$

Definition 2.4

A fuzzy set $\tilde{A}$ is said to be **normal** if and only if hgt $(\tilde{A})=1$ and **convex** if and only if $\forall x,y\in\tilde{A},\forall\alpha\in[0,1]$

$$\eta_{\tilde{A}}(\lambda x+(1-\lambda)y\geq\min\{\eta_{\tilde{A}}(x),\eta_{\tilde{A}}(y)\}$$

Definition 2.5

A fuzzy set $\tilde{A}$ in the universe X is a **fuzzy number** if and only if it satisfies the following conditions

1) X = R
2) $\tilde{A}$ is normal
3) $\tilde{A}$ is convex
4) The membership function $\eta_{\tilde{A}}$ is piecewise continuous.
5) There exists one and only one x ∈ R such that $\eta_{\tilde{A}}(x)=1.$

Definition 2.6

A **fuzzy** number $\tilde{A}$ is said to be triangular fuzzy number iff there exist real numbers a<b<c such that

$$\eta_{\tilde{A}}(x)=\begin{cases}\left(\dfrac{x-a}{x-b}\right), a\le x\le b\\ \left(\dfrac{c-x}{c-b}\right), b\le x\le c\\ 0, Otherwise\end{cases}$$

Denoted by its membership function

$\tilde{A}=(a/b/c)$ or $\tilde{A}=(a,b,c)$

Definition 2.7

A fuzzy number $\tilde{M}$ is said to be "**L-R fuzzy number**" if and only if there exists three real numbers m, a > 0, b > a and two positive continuous and decreasing functions L and R, from R → [0,1] such that : L (0) = R (0) = 1.

$$L(1)=0, L(x)>0, \lim_{x\to\infty} L(x)=0$$

$$R(1)=0, R(x)>0, \lim_{x\to\infty} R(x)=0$$

$$\eta_{\tilde{M}}(x)=\begin{cases}L\left(\dfrac{m-x}{a}\right) \text{ if } x\in[m-a,m]\\ R\left(\dfrac{x-m}{b}\right) \text{ if } x\in[m,m+b]\\ 0 \qquad \text{otherwise}\end{cases}$$

The L-R representation of the fuzzy number $\tilde{M}$ is $\tilde{M}=\langle m,a,b\rangle_{LR}$ .where m is called the mean value (the mode (or) the modal value) of $\tilde{M}$, a and b are called respectively, the left spread and right spread of $\tilde{M}$. conventionally, $\langle m,0,0\rangle_{LR}$ is the ordinary real numbers m; also called the fuzzy singleton. According to,

$$\text{Supp}(\tilde{A})=\{x\in E/\eta_{\tilde{A}}(x)>0\}$$

⇒Support of $\tilde{M}$ is the following open internal

$$\text{Supp}\left(\tilde{M}\right) =]m-a, m[\cup]m, m+b[$$

$$=]m-a, m+b[$$

$$\text{Supp}\ \tilde{M} =]m-a, m+b[$$

Remark:

i) If L = R, then $\tilde{M} = < m, a, b > L_R$ is said to be semi symmetric.

ii) If $\tilde{M} = < m, a, b > L_R$ is semi symmetric and then $\tilde{M} = < m, a, b > L_R$ is said to be symmetric.

Definition 2.8:

Alpha cuts & Interval arithmetic:

An **α-cut**(or)**α-level set** of a fuzzy set $A \subseteq X$ is an ordinary set $A_\alpha \subseteq X$, such that:

$$A_\alpha = \{\mu_A(x) \geq \alpha, \forall x \in X\}$$

Arithmetic operations

Let $[a_1,b_1]$ and $[a_2,b_2]$ be two closed, bounded intervals of real numbers. If denotes addition, Subtraction, multiplication, or division, then

$$[a_1,b_1]*[a_2,b_2] = [k,l] \text{ Where} [k,l] = \{a*b / a_1 \leq a \leq b_1, a_2 \leq b \leq b_2\}$$

(i) $[a_1,b_1]+[a_2+b_2]=[a_1+a_2, b_1+b_2]$

(ii) $[a_1,b_1]-[a_2-b_2]=[a_1-b_2, b_1-a_2]$

(iii) $[a_1,b_1].[a_2,b_2]=[\max(a_1a_2, a_1b_2, b_1, a_2, b_1\ b_2);\ \min(a_1a_2, a_1b_2, b_1a_2, b_1b_2)]$

(iv) $\dfrac{[a_1,b_1]}{[a_2,b_2]} = \left[\min\left(\dfrac{a_1}{a_2}, \dfrac{a_1}{b_2}, \dfrac{b_1}{a_2}, \dfrac{b_1}{b_2}\right); \max\left(\dfrac{a_1}{a_2}, \dfrac{a_1}{b_2}, \dfrac{b_1}{a_2}, \dfrac{b_1}{b_2}\right)\right]$

Definition: 2.9

Let the two triangular fuzzy numbers be $\tilde{M} \approx (a_1,a_2,a_3)$ and $\tilde{N} \approx (b_1,b_2,b_3)$ and then the arithmetic operations on triangular fuzzy numbers be given as follows.

(A) Addition

$$\tilde{M}+\tilde{N} \approx (a_1,a_2,a_3)+(b_1,b_2,b_3)$$

$$\tilde{M}+\tilde{N} \approx (m_1,\alpha_1,\beta_1)+(m_2,\alpha_2,\beta_2)$$

$$\tilde{M}+\tilde{N} \approx (m_1+m_2, \max\{\alpha_1,\alpha_2\}, \max\{\beta_1,\beta_2\})$$

(B) Subtraction

$$\tilde{M}-\tilde{N} \approx (a_1,a_2,a_3)-(b_1,b_2,b_3)$$

$$\tilde{M}-\tilde{N} \approx (m_1,\alpha_1,\beta_1)-(m_2,\alpha_2,\beta_2)$$

$$\tilde{M}-\tilde{N} \approx \{(m_1-m_2), \max\{\alpha_1,\alpha_2\}, \max(\beta_1,\beta_2)\}$$

(C) Multiplication

$$\tilde{M}.\tilde{N} \approx (a_1,a_2,a_3).(b_1,b_2,b_3)$$

$$\tilde{M}.\tilde{N} \approx (m_1,\alpha_1,\beta_1).(m_2,\alpha_2,\beta_2)$$

$$\tilde{M}.\tilde{N} \approx \left((m_1.m_2), \max\{\alpha_1, \alpha_2\}, \max\{\beta_1, \beta_2\} \right)$$

(D) Division

$$\frac{\tilde{M}}{\tilde{N}} = \frac{(a_1, a_2, a_3)}{(b_1, b_2, b_3)}$$

$$\frac{\tilde{M}}{\tilde{N}} = \frac{(m_1, \alpha_1, \beta_1)}{(m_2, \alpha_2, \beta_2)}$$

$$\frac{\tilde{M}}{\tilde{N}} = \left(\frac{m_1}{m_2}, \max\{\alpha_1, \alpha_2\}, \max\{\beta_1, \beta_2\} \right)$$

L-R Method description

Let us consider the queue $M/E_K/1$. Suppose the arrival rate and service rate are all triangular fuzzy numbers denoted respectively $\tilde{\lambda} = (\lambda_1, \lambda_2, \lambda_3)$ and $\tilde{\mu} = (\mu_1, \mu_2, \mu_3)$. The fuzzy information transmitted through $\tilde{\lambda}$ and $\tilde{\mu}$ in the system affect L_q and w_q, which becomes fuzzy numbers $\tilde{L}_q$ and $\tilde{W}_q$.

The model becomes in this case a fuzzy queue and it is denoted by $FM/FE_K/1$, The inter arrival and service time are fuzzy natured. To determine its fuzzy performance measures, L-R method can be used. The fuzzy rates $\tilde{\lambda}$ and $\tilde{\mu}$ in L-R representation, and replaces them to obtain L-R fuzzy numbers L_q, W_q, L_s and W_s using suitable formulas.

3. Mathematical model

Consider a queuing system in k phases single server facility, denoted by $FM/FE_K/1$ in which arrivals occur as poisson process with fuzzy rate $\tilde{\lambda}$ and service time according to Erlang's k distribution with fuzzy rate $\tilde{\mu}$. It is assumed that the service discipline is first in first out and the system capacity is infinite.

Expected "Number of customers" and their "waiting time" in a queue using fuzzy environment:

Let the triangular fuzzy number be $\tilde{\lambda} = (a_1, a_2, a_3)$ and $\tilde{\mu} = (b_1, b_2, b_3)$ respectively with $a_3 < b_1$;

In fuzzy queuing theory, the number of customers, waiting time of the customers, arrival rateand service rate are taken as non-negative values. Let Lq be the "number of customers" in the fuzzy queue and it can be computed by replacing λ and μ by $\tilde{\lambda}$ and $\tilde{\mu}$ respectively.

i) Expected number of customers in the queue

The "number of customers" is denoted by Lq and is given by $Lq = \frac{k+1}{2k} \left\{ \frac{\tilde{\lambda}^2}{\tilde{\mu}(\tilde{\mu} - \tilde{\lambda})} \right\}$

Let $\tilde{\lambda} = (m_1, \alpha_1, \beta_1)$ and $\tilde{\mu} = (m_2, \alpha_2, \beta_2)$ where m_1 is the middle number, α_1 is the left Hand derivation, β_1 is the right hand derivation of λ.

$$Lq = \left(\frac{k+1}{2k}\right)\left\{\frac{(m_1,\alpha_1,\beta_1).(m_1,\alpha_1,\beta_1)}{(m_2,\alpha_2,\beta_2).(m_2,\alpha_2,\beta_2)-(m_1,\alpha_1,\beta_1)}\right\}$$

$$Lq = \left(\frac{k+1}{2k}\right)\left\{\left(\frac{m_1^2}{m_2^2-m_1m_2}\right), \max\{\alpha_1,\alpha_2\}, \max\{\beta_1,\beta_2\}\right\}$$

ii) Waiting time in the queue

$$Wq = \left(\frac{k+1}{2k}\right)\left(\frac{\tilde{\lambda}}{\tilde{\mu}(\tilde{\mu}-\tilde{\lambda})}\right)$$

$$Wq = \left(\frac{k+1}{2k}\right)\left\{\frac{(m_1,\alpha_1,\beta_1)}{(m_2,\alpha_2,\beta_2)\left[(m_2,\alpha_2,\beta_2)-(m_1,\alpha_1,\beta_1)\right]}\right\}$$

$$Wq = \left(\frac{k+1}{2k}\right)\left[\frac{m_1}{m_2^2-m_1m_2}, \max\{\alpha_1,\alpha_2\}, \max\{\beta_1,\beta_2\}\right]$$

iii) Expected number of customers in the system

$$L_s = Lq + \frac{\tilde{\lambda}}{\tilde{\mu}}$$

$$L_s = \left(\frac{k+1}{2k}\right)\left(\frac{m_1^2}{m_2^2-m_1m_2}, \max\{\alpha_1,\alpha_2\}, \max\{\beta_1,\beta_2\}\right) + \left[\frac{(m_1,\alpha_1,\beta_1)}{(m_2,\alpha_2,\beta_2)}\right]$$

$$L_s = \left(\frac{k+1}{2k}\right)\left\{\left(\frac{m_1m_2^2}{m_2^3-m_1m_2^2}\right), \max\{\alpha_1,\alpha_2\}, \max\{\beta_1,\beta_2\}\right\}$$

iv) Waiting time in the system

$$Ws = Wq + \frac{1}{\tilde{\mu}}$$

$$W_s = \left(\frac{k+1}{2k}\right)\left(\frac{m_1}{m_2^2-m_1m_2}, \max\{\alpha_1,\alpha_2\}, \max\{\beta_1,\beta_2\}\right) + \frac{1}{(m_2,\alpha_2,\beta_2)}$$

$$W_s = \left(\frac{k+1}{2k}\right)\left\{\left(\frac{m_2^2}{m_2^3-m_1m_2^2}\right), \max\{\alpha_1,\alpha_2\} \max\{\beta_1,\beta_2\}\right\}$$

4. Solution procedure

Consider a queuing system in k-phases single server facility denoted by $FM/FE_K/1$ in which arrivals occur as poisson process with fuzzy rate $\tilde{\gamma}$ and service time according to Erlang's k distribution with fuzzy rate $\tilde{\mu}$. The service discipline is first in first out and the system capacity is infinite.

$$\left(FM / FE_k / 1 : \infty / FIF0\right)$$

Suspicions

i) Infinite capacity FM/FE$_K$/1, Queuing model with one server.
ii) Inter arrival time follows poisson distribution
iii) Service (Erlang) time follows exponential distribution
iv) Arrival rate, service rate are fuzzy numbers.

5. Numerical example

(I) Alpha – cuts method solution

In order to justify the benefits of this new method, this section compares it through the following problem with an alpha-cuts method.

Let the triangular fuzzy number be $\tilde{\lambda} = (20, 21, 22)$ and $\tilde{\mu} = (23, 24, 25)$ The service consists of 3-phases each arrival and service rate are triangular fuzzy members denoted by $\tilde{\lambda} = (20, 21, 22)$ and $\tilde{\mu} = (23, 24, 25)$

The confidence interval at α are [α+20, - α+22] and [α+23, - α+25]

i) Expected number of customers in the queue

$$L_{q_\alpha} = (0.66667)\left\{\frac{[\alpha+20][-\alpha+22].[\alpha+20][-\alpha+22]}{[\alpha+23,-\alpha+25].[(\alpha+23,-\alpha+25)-(\alpha+20,-\alpha+22)]}\right\}$$

$$L_{q_\alpha} = (0.66667)\left\{\frac{(\alpha+20)(-\alpha+22).(\alpha+20)(-\alpha+22)}{(\alpha+23,-\alpha+25).(2\alpha+1,-2\alpha+5)}\right\}$$

$$L_{q_\alpha} = (0.66667)\left[\frac{\alpha^2+40\alpha+400}{2\alpha^2-55\alpha+125}, \frac{\alpha^2-44\alpha+484}{2\alpha^2+47\alpha+23}\right]$$

$\therefore$ We obtain triangular fuzzy number as follows:

$$\therefore \tilde{\mathbf{N}} = L_{q_\alpha} = \left(\mathbf{2.13334 / 4.08335 / 14.02906}\right)$$

The membership function of L_q is defined as follows

$$\eta_{\tilde{N}}(x) = \begin{cases} \dfrac{x-2.13334}{1.95001} & \text{if } 2.13334 \le x \le 4.08335 \\ \dfrac{14.02906-x}{9.94571} & \text{if } 4.08335 \le x \le 14.02906 \\ 0 & \text{otherwise} \end{cases}$$

The number of customers is approximately between 2.13334 and 14.02906

ii) waiting time in the queue

$$W_{q_\alpha} = \left(\frac{k+1}{2k}\right)\left(\frac{\tilde{\lambda}}{\tilde{\mu}(\tilde{\mu}-\tilde{\lambda})}\right),$$

Let $\tilde{\gamma} = (20, 21, 22)$ & $\tilde{\mu} = (23, 24, 25)$ be the triangular fuzzy numbers. The confidence interval at α are [α+20,-α+22] and [α+23, - α+25]

$$W_{q_\alpha} = (0.66667)\left\{\frac{[\alpha+20, -\alpha+22]}{[\alpha+23, -\alpha+25].[(\alpha+23, -\alpha+25)-(\alpha+20, -\alpha+22)]}\right\}$$

$$= (0.66667)\left\{\frac{[\alpha+20, -\alpha+22]}{[\alpha+23, -\alpha+25].[2\alpha+1, -2\alpha+5]}\right\}$$

$$W_{q_\alpha} = (0.66667)\left(\frac{\alpha+20}{2\alpha^2-55\alpha+125}, \frac{-\alpha+22}{2\alpha^2+47\alpha+23}\right)$$

$$\therefore \mathbf{\tilde{T}} = W_{q_\alpha} = (\mathbf{0.10667 / 0.19445 / 0.63768})$$

$$\text{Hence } \eta_{\tilde{T}}(x) = \begin{cases} \dfrac{x-0.10667}{0.08778} & \text{if } 0.10667 \le x \le 0.19445 \\ \dfrac{0.63768-x}{0.44323} & \text{if } 0.19445 \le x \le 0.63768 \\ 0 & \text{otherwise} \end{cases}$$

iii) Number of customers in the system

$$L_{s_\alpha} = L_{q_\alpha} + \frac{\tilde{\lambda}}{\tilde{\mu}}$$

$$L_{s_\alpha} = (0.66667)\left[\frac{\alpha^2+40\alpha+400}{2\alpha^2-55\alpha+125}, \frac{\alpha^2-44\alpha+484}{2\alpha^2+47\alpha+23}\right] + \frac{[\alpha+20, -\alpha+22]}{[\alpha+23, -\alpha+25]}$$

$$\therefore \mathbf{L}_{s_\alpha} = (\mathbf{2.93334 / 4.95835 / 14.98558})$$

iv) Waiting time in the system

$$W_{s_\alpha} = W_{q_\alpha} + \frac{1}{\tilde{\mu}}$$

$$= (0.66667)\left\{\frac{\alpha+20}{2\alpha^2-55\alpha+125}, \frac{-\alpha+22}{2\alpha^2+47\alpha+23}\right\} + \left[\frac{1}{(\alpha+23, -\alpha+25)}\right]$$

$$W_{s_\alpha} = (0.66667)\left\{\frac{\alpha+20}{2\alpha^2-55\alpha+125}, \frac{-\alpha+22}{2\alpha^2+47\alpha+23}\right\} + \left[\frac{1}{-\alpha+25}, \frac{1}{\alpha+23}\right]$$

$$\therefore \mathbf{W}_{s_\alpha} = (\mathbf{0.14667 / 0.23612 / 0.68116})$$

(II) L – R Method Solution

Such triangular fuzzy number is often noted $\tilde{A}=(a,b,c)$ or $\tilde{A}=(a/b/c)$. According to the definitions, a triangular fuzzy number $\tilde{A}=(a/b/c)$ is always an L-R fuzzy number. In L-R representation, it can be written for L(x) = R(x) = max (0, 1-x).

(i)Let the triangular fuzzy numbers be $\tilde{\lambda}=(20,21,22)$ and $\tilde{\mu}=(23,24,25)$

(ii) Determine the triangular fuzzy number $\tilde{\lambda}=(20,21,22)$ and $\tilde{\mu}=(23,24,25)$ is of the form (m, α, β)

$$\tilde{\lambda}=\langle 21,1,1\rangle \mathrm{LR}$$

$$\tilde{\mu}=\langle 24,1,1\rangle \mathrm{LR}$$

a) The number of customers in the queue.

$$L_q=\left(\frac{k+1}{2k}\right)\left(\frac{\tilde{\lambda}^2}{\tilde{\mu}\left(\tilde{\mu}-\tilde{\lambda}\right)}\right)$$

$$\therefore L_q=(0.66667)\left\{\frac{\langle 21,1,1\rangle_{LR}\langle 21,1,1\rangle_{LR}}{\langle 24,1,1\rangle_{LR}\left[\langle 24,1,1\rangle_{LR}-\langle 21,1,1\rangle_{LR}\right]}\right\}$$

$$\therefore Lq=\langle 4.08335, 0.66667, 0.66667\rangle_{LR}$$

$$\therefore L_q=(3.41668/4.08335/4.75002)$$

b) Waiting time in the queue

$$W_q=\left(\frac{k+1}{2k}\right)\left(\frac{\tilde{\lambda}}{\tilde{\mu}\left(\tilde{\mu}-\tilde{\lambda}\right)}\right),$$

$$=(0.66667)\left\{\frac{\langle 21,1,1\rangle_{LR}}{\langle 24,1,1\rangle_{LR}.\left[\langle 24,1,1\rangle_{LR}-\langle 21,1,1\rangle_{LR}\right]}\right\}$$

$$W_q=\langle 0.19445, 0.6667, 0.6667\rangle$$

$$\therefore W_q=(0.47222/0.19445/0.86112)$$

c) Number of customers in the system

$$L_s=L_q+\frac{\tilde{\lambda}}{\tilde{\mu}}$$

$$L_s=\langle 4.08335, 0.6667, 0.66667\rangle_{LR}+\frac{\langle 21,1,1\rangle_{LR}}{\langle 24,1,1\rangle_{LR}}$$

$$L_s=\langle 4.95835,1,1\rangle_{LR}$$

$$\therefore L_s = \left(3.95835/\,4.95835/\,5.95835\right)$$

d) **Waiting time in the system**

$$W_s = W_q + \frac{1}{\tilde{\mu}}$$

$$W_s = \langle 0.19445, 0.66667, 0.66667 \rangle_{LR} + \frac{1}{\langle 24,1,1 \rangle_{LR}}$$

$$W_s = \langle 0.23612, 1, 1 \rangle_{LR}$$

$$\therefore W_s = (0.76388 / 0.23612 / 0.76388)$$

In this paper, the performance Measures of FM/FE$_K$/1 queuing model are computed using L-R Method. This method has a superiority of being comfortable and supple compared to the well-known and called Alpha-cuts method.

Result

	α-Cut method	**L-R method**
1.	$L_{q_\alpha} = (2.13334, 4.08335, 14.02906)$	$L_q = (3.41668, 4.08335, 4.75002)$
2.	$W_{q_\alpha} = (0.10667, 0.19445, 0.63768)$	$W_q = (0.47222, 0.19445, 0.86112)$
3.	$L_{s_\alpha} = (2.9334, 4.95835, 14.98558)$	$L_s = (3.95835, 4.95835, 5.95835)$
4.	$W_{s_\alpha} = (0.14667, 0.23612, 0.68116)$	$W_s = (0.76388, 0.23612, 0.76388)$

In this paper, we suggested an analytical approach to solve fuzzy queuing problem which gives L_q − the expected "number of customers" and W_q − the "waiting time of customers" in the queue by α-cut method and by L-R method. Where as we have got L_s − the number of customers and W_s − the waiting time of customers in the system computing by both methods in a successful way. Therefore we found that the result obtained by our method is much more optimized than the existing one.

6. Conclusion

In this paper, the L-R representation has been used to derive performance measures of the queuing model with the help of this method, the expected number of customers and the average waiting time of FM/FE$_K$/1 model are successfully computed and results are found in L-R representation. Under this representation, fuzzy results give many pieces of informations than the crisp ones. This new method presents three major advantages, it is short, convenient and flexible and approximate method for best explanatory results.

References

1. J.J. Buckley, Elementry queuing theory based on possibility theory, Fuzzy Sets and Systems, 37(1990) 43-52.
2. S.P. Chen., "Parametric non-linear programming approach to fuzzy queues with bulk service", European Journal of Operational Research 163, 434-444, 2004.
3. S.P. Chen, "A Mathematics Programming approach to the machine interference problem with

fuzzy parameters, Applied Mathematics and Computation 174, 374-387, 2006.

4. Donald Gross.," Fundamentals of queuing theory". A wiley-Interscience Publication, John wiley & sons, INC, Third edition.

5. B. Kalpana and N. Anusheela, "Analysis of a single server Non-preemptive fuzzy priority Queue using L-R method", ARPN, journal of Engineering and applied sciences, Vol 13, No. 23, Dec 2018, (9306-9310)D.S. Negi, E.S. Lee, "Analysis and simulation of fuzzy queues", fuzzy sets and systems, 46(1992). (321-320)

6. R.J.Li, E.S. Lee, "Analysis of fuzzy queues", proc. NAFIPS 1998, (1988) 158-162.

7. R.J.Li, E.S. Lee, "Analysis of fuzzy queues, computers and mathematics with Applications", 17 (1989) 1143-1147.

8. J.P. Mukeba Kanyinda, R. Mabela Makengo Matendo, B.Ulungu ekunda lukata, "computing fuzzy queuing performance measures by L-R method", journal of fuzzy set valued Analysis 2015 No. 1 (2015) 57-67.

9. J.P. Mukeba, "Applications of L-R method to single server fuzzy retrial queue with patient customers", journal of pure and applied mathematics Advances and Applications volume 16, No 1, 2016 (43-59)

10. S.Narayanamoorthy.,A.Anuja.,J.V.Brainy.,Daekook Kang.,S,Maheswari.,"Analysis of fuzzy queuing performance measures by the L-R method with Erlang service model",AIP Publishing,2020

11. D.S. Negi, E.S. Lee, "Analysis and simulation of fuzzy queues", fuzzy sets and systems, 46(1992). (321-320).

12. W. Ritha, L. Robert, "Application of fuzzy set theory to retrial queues", International journal of Algorithms, computing and Mathematics, Q(4) (2009) 9-18

13. W. Ritha, S.B. Menon, "Fuzzy n policy queues with infinite capacity", journal of physical sciences, 15(2011) (73-82)

14. W. Ritha S. Josephine Vinnarasi "Analysis of priority queuing models : L-R method", Annals of pure and Applied Mathematics, Vol 15, No. 2, 2017, 271-276

15. S. Shanmugasundaram, S. Thamotharan, M. Ragapriya "A study on single server fuzzy queuing model using DSW Algorithm", International journal of Latest Trends in Engineering and Technology, Vol 6, issue 1, September 2015, 162-169

16. M. Shanmugasundari and S. Aarthi.,"A different approach to solve fuzzy queuing theory" AIP conference procedings 2277 090011(1-7), 6 Nov 2020 1st International conference on Mathematical Techniques and Applications.

17. Zadeh L. A., "Fuzzy sets as basis for a theory of Possibility", Fuzzy Sets and Systems, (1), 3-28, 1978.

18. H.J. Zimmermann, "Fuzzy set theory and its applications" Springer science + Businessmedia, New york, fourth edition, (2001).