



Resolving Sets and Dimension in Bidiakis Cube and Durer Graphs

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Abstract

In this research paper, presented the idea resolving set using dominating set as a base set in special graphs like Bidiakis cube, Durer graph, Golomb graph and etc. In a graph $G = (V, E)$, The code of vertex v with respect to the ordered set $W = \{w_1, w_2, w_3, \dots, w_k\} \subseteq V(G)$ is defined by $C_w(v) = (d(v, w_1), d(v, w_2), \dots, d(v, w_k))$. The set W is so-called a resolving set for G if different nodes have different codes with respect to W . A resolving set having a minimum number of nodes is a minimum resolving set or a basis for G . The (metric) dimension $\text{Dim}(G)$ is the quantity of nodes in a basis for $G = (V, E)$.

KeyWords: Graphs, Code, Resolving set, Code matrix, metric

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Introduction

Domination in graph theory is a natural model for many location problems in operations research. As an example, consider the following fire station problem. Suppose a county has decided to build some fire stations, to serve all of the towns in the county. They are to be placed in some towns so that every town either has a fire station or is a neighbor of a town which has a fire station. To save money, the county wants to build the minimum number of fire stations satisfying the above requirements.

A graph is an ordered pair $G = (V, E)$, where V is a non-empty finite set, called the set of vertices of G , and E is a set of unordered pairs (2-element subsets) of V , called the edges of G . If $xy \in E$, x and y are called adjacent and they are incident with the edge xy .

For graph theoretic jargon we refer to Chartrand and Lesniak. The main focus of this paper is a generalization of the concept of domination in graphs. For an outstanding dealing of fundamentals

For survey of several advanced topics in domination we refer to the book edited by Haynes et al.

The code of vertex v with respect to the ordered set $W = \{w_1, w_2, w_3, \dots, w_k\} \subseteq V(G)$

in the graph $G(V, E)$ is defined by $C_w(v) = (d(v, w_1), d(v, w_2), \dots, d(v, w_k))$. The set W is termed a resolving set for G if different nodes have different codes with respect to W . A resolving set containing a minimum number of vertices is a minimum resolving set or a basis for G . The (metric) dimension $\text{dim}(G)$ is the number of vertices in a basis for G .

The resolving matrix of the finite graph $G(V, E)$ with respect to the ordered resolving set $R = \{v_1, v_2, v_3, \dots, v_k\} \subseteq V(G)$ is defined by the $C_R(G) = [a_{ij}]$, a_{ij} denotes the distance between the nodes v_i and v_j .

of domination we refer to the book by Haynes et al.

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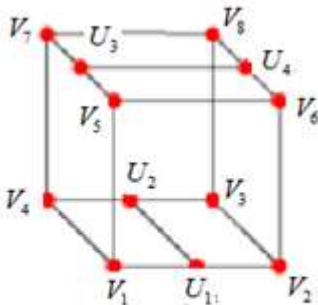
In $C_R(G) = [a_{ij}]$ rows contains the elements of $V(G) - R$ and column contains the elements in R . The resolving matrix is defined by

$$C_R(G) = \begin{matrix} & v_{r_1} & v_{r_2} & v_{r_3} & \dots & v_{r_k} \\ \begin{matrix} v_{c_1} \\ v_{c_2} \\ v_{c_3} \\ \vdots \\ v_{c_n} \end{matrix} & \begin{bmatrix} d(v_{c_1}, v_{r_1}) & d(v_{c_1}, v_{r_2}) & d(v_{c_1}, v_{r_3}) & \dots & d(v_{c_1}, v_{r_k}) \\ d(v_{c_2}, v_{r_1}) & d(v_{c_2}, v_{r_2}) & d(v_{c_2}, v_{r_3}) & \dots & d(v_{c_2}, v_{r_k}) \\ d(v_{c_3}, v_{r_1}) & d(v_{c_3}, v_{r_2}) & d(v_{c_3}, v_{r_3}) & \dots & d(v_{c_3}, v_{r_k}) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ d(v_{c_n}, v_{r_1}) & d(v_{c_n}, v_{r_2}) & d(v_{c_n}, v_{r_3}) & \dots & d(v_{c_n}, v_{r_k}) \end{bmatrix} \end{matrix}$$

$v_{r_i} \in R$ where $i = 1, 2, \dots, k$ and
 $v_{c_j} \in V(G) - R$ where $j = 1, 2, \dots, n$.

Main Results

Bidiakis Cube: Bidiakiscube is a 3-regular graph with 12 vertices and 18 edges.



$G(V, E)$: Bidiakis cube

The 12-node graph containing of a cube in which two opposite surfaces (say, top and bottom) be necessary edges drawn through them which join the midpoints of opposite sides of the faces in such a way that the orientation of the edges added on top and bottom are perpendicular to each other.

Theorem 2.1. Let $G(V, E)$ is a Bidiakis Cube. Then the minimal resolving set $R = \{U_i, U_j, U_k\}$ where $U_i, U_j, U_k \in V(G)$, and the dimension of Bidiakis Cube $G(V, E)$ is $Dim(G) = 3$.

Proof: Let $G(V, E)$ is a Bidiakis Cube. Let we assume the minimal dominating set $D = \{U_1, U_2, U_3, U_4\}$. Let U_1, U_2, U_3, U_4 containing the vertices in two opposite faces have edges drawn across them which connect the centres of opposite sides of the

faces. Let $D = \{a, b, c, d\}$, it dominates the corner vertices of the Bidiakis Cube. Therefore U_1, U_2, U_3, U_4 is a dominating set of Bidiakis Cube.

The resolving matrix of $C_D(G)$ is

$$C_D(G) = \begin{matrix} & U_1 & U_2 & U_3 & U_4 \\ \begin{matrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ U_1 \end{matrix} & \begin{bmatrix} 1 & 2 & 2 & 3 \\ 1 & 2 & 3 & 2 \\ 2 & 1 & 3 & 2 \\ 2 & 1 & 2 & 3 \\ 2 & 3 & 1 & 2 \\ 2 & 3 & 2 & 1 \\ 3 & 2 & 1 & 2 \\ 3 & 2 & 2 & 1 \end{bmatrix} \end{matrix}$$

Note that code of the of every pair of vertices are distinct i.e. $C_D(v_i) \neq C_D(v_j)$ for every $v_i, v_j \in V(G)$. Therefore $D = \{U_1, U_2, U_3, U_4\}$ is the resolve dominating set of Bidiakis Cube $G(V, E)$. Now we check $D = \{U_1, U_2, U_3, U_4\}$ is the minimal resolve dominating set of Bidiakis Cube $G(V, E)$.

Assume $D = \{U_1, U_2, U_3, U_4\}$ is a minimal resolve dominating set of Bidiakis Cube. There exist a set $R = (D - \{U_i\})$ where $U_i \in V(G)$ is the vertices in two opposite faces have edges drawn across them which connect the centres of opposite sides of the faces. Let $R = (D - \{U_1\})$ the resolving matrix of $C_R(G)$ is

$$C_R(G) = \begin{matrix} & U_2 & U_3 & U_4 \\ \begin{matrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ U_1 \end{matrix} & \begin{bmatrix} 2 & 2 & 3 \\ 2 & 3 & 2 \\ 1 & 3 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & 3 & 3 \end{bmatrix} \end{matrix}$$

Note that code of the of every pair of vertices are distinct i.e. $C_R(v_i) \neq C_R(v_j)$ for every $v_i, v_j \in V(G)$.



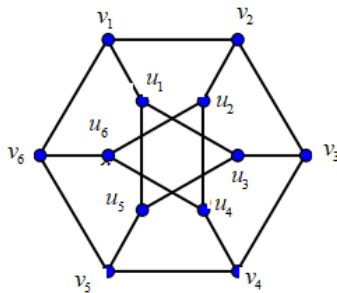
Therefore $R = (D - \{U_i\})$ is the resolving set of Bidiakis Cube $G(V, E)$ but not dominating set. Since $D = \{U_1, U_2, U_3, U_4\}$ is a minimal dominating set of Bidiakis Cube. Assume $R = (D - \{U_i\})$ is not a minimal resolving set of Bidiakis Cube. There exist a set $R' = (D - \{U_i, U_j\})$ where $U_i, U_j \in V(G)$ is the vertices in two opposite faces have edges drawn across them which connect the centres of opposite sides of the faces. From the resolving matrix of $C_{R'}(G)$ we observe the code of $V_i, V_j \in V(G)$ are not distinct, i.e. $C_{R'}(v_i) = C_{R'}(v_j)$ for some $V_i, V_j \in V(G)$, Therefore R' is not a resolving set of Bidiakis Cube $G(V, E)$. Hence $R = (D - \{U_i\})$ is a minimal resolving set of Bidiakis Cube.

The set R is a minimal resolving set or a basis of Bidiakis Cube $G(V, E)$. Therefore dimension of Bidiakis Cube $G(V, E)$ is cardinality of the basis R .

$$\begin{aligned} Dim(G) &= |R| = |D - \{U_i\}| \\ &= |D| - |U_i| \\ &= |\{U_1, U_2, U_3, U_4\}| - |U_i| \end{aligned}$$

$$Dim(G) = 3$$

Durer graph: The Durer graph is an undirected cubic graph with 12 vertices and 18 edges.



$G(V, E)$: Durer graph

Theorem 2.2 Let $G(V, E)$ is a Durer graph. Then the minimal resolving set $R = (D - \{v_{(i+6,3)}\})$, where D is the minimal resolving set of Durer graph $G(V, E)$. The dimension of Durer graph $G(V, E)$ is $Dim(G) = 3$.

Proof: Let $G(V, E)$ is a Durer graph. Let we assume

$D = \{u_i, u_{(i+6,1)}, v_i, v_{(i+6,3)}\}$ is a minimal dominating set of Durer graph. The subset $\{u_i, u_{(i+6,1)}\}$ dominates remaining inner vertices of Durer graph and the subset $\{v_i, v_{(i+6,3)}\}$ remaining outer vertices of Durer graph. Note that $D - \{x\}$, is not a dominating set. If $x \in \{u_i, u_{(i+6,1)}\}$ there exist an inner vertex $u_j \in V(G)$ not dominated by D . Therefore $D = \{u_i, u_{(i+6,1)}, v_i, v_{(i+6,3)}\}$ is a minimal dominating set of Durer graph $G(V, E)$. The resolving matrix of $C_D(G)$ is

$$C_D(G) = \begin{matrix} & u_1 & u_2 & v_1 & v_4 \\ \begin{matrix} u_3 \\ u_4 \\ u_5 \\ u_6 \\ v_2 \\ v_3 \\ v_5 \\ v_6 \end{matrix} & \begin{bmatrix} 1 & 2 & 2 & 2 \\ 3 & 1 & 3 & 1 \\ 1 & 3 & 2 & 2 \\ 2 & 1 & 2 & 2 \\ 2 & 1 & 1 & 2 \\ 2 & 2 & 2 & 1 \\ 2 & 3 & 2 & 1 \\ 2 & 2 & 1 & 3 \end{bmatrix} \end{matrix}$$

Note that code of the of every pair of vertices are distinct i.e. $C_R(v_i) \neq C_R(v_j)$ for every $v_i, v_j \in V(G)$.

Therefore $D = \{u_i, u_{(i+6,1)}, v_i, v_{(i+6,3)}\}$ is resolving set of Durer graph $G(V, E)$. Assume $R = (D - \{v_{(i+6,3)}\})$ is a resolving set of Durer graph. The code matrix of $C_R(G)$ is

$$C_R(G) = \begin{matrix} & u_1 & u_2 & v_1 \\ \begin{matrix} u_3 \\ u_4 \\ u_5 \\ u_6 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{matrix} & \begin{bmatrix} 1 & 2 & 2 \\ 3 & 1 & 3 \\ 1 & 3 & 2 \\ 2 & 1 & 2 \\ 2 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 2 & 3 \\ 2 & 3 & 2 \\ 2 & 2 & 1 \end{bmatrix} \end{matrix}$$

Note that code of the of every pair of vertices are distinct i.e. $C_R(v_i) \neq C_R(v_j)$ for every $v_i, v_j \in V(G)$.

Therefore $R = (D - \{x\})$ is the resolving set but not



a dominating of Durer graph $G(V, E)$. Assume $R = (D - \{x\})$ is not a minimal resolving set of Durer graph. There exist a set $R' = (D - \{x_i, x_j\})$ is the resolving set of Durer graph $G(V, E)$. From the resolving matrix of $C_R(G)$ we observe the code of $v_i, v_j \in V(G)$ are same, i.e. $C_{R'}(v_i) = C_{R'}(v_j)$ for some $v_i, v_j \in D$, Therefore R' is not a resolving of Durer graph $G(V, E)$. Hence $R = (D - \{v_{(i+6,3)}\})$ is a minimal resolving set of Durer graph.

The set R is a minimal resolving set or a basis of Durer graph $G(V, E)$. Therefore dimension of Durer graph $G(V, E)$ is cardinality of the basis R .

$$\begin{aligned} Dim(G) &= |R| = |D - \{v_{(i+6,3)}\}| \\ &= |D| - |v_{(i+6,3)}| \\ &= |u_i, u_{(i+6,1)}, v_i, v_{(i+6,3)}| - |v_{(i+6,3)}| \end{aligned}$$

$$Dim(G) = 3$$

Note: In a Durer graph $G(V, E)$, every minimal dominating set is a minimal resolve dominating set of $G(V, E)$,

Theorem 2.3 Let $G(V, E)$ is a Petersen graph. Then the minimal resolving set $R = (D \cup \{v_{(i+5,3)}\})$

where D is a minimal dominating set and $v_{(i+5,3)}$ is a vertex on the pentagon of Petersen graph $G(V, E)$. The dimension of Petersen graph $G(V, E)$ is $Dim(G) = 4$.

Proof: Let $G(V, E)$ is a Petersen graph. Let $D = \{u_i, u_{(i+5,4)}, v_{(i+5,2)}\}$ is a minimal dominating set of Petersen graph. The vertices $u_i, u_{(i+5,4)}$ dominates remaining vertices in inner region and also dominates $v_i, v_{(i+5,4)}$ in inner region. The vertex $v_{(i+5,2)}$ dominates $v_{(i+5,1)}$ & $v_{(i+5,3)}$. Therefore $D = \{u_i, u_{(i+5,4)}, v_{(i+5,2)}\}$ is a minimal dominating set of Petersen graph $G(V, E)$. Let $D = \{u_1, u_5, v_3\}$ is a minimal dominating set of Petersen graph $G(V, E)$. The code matrix of $C_D(G)$ is

$$C_D(G) = \begin{matrix} & u_1 & u_5 & v_3 \\ u_2 & \begin{bmatrix} 2 & 1 & 2 \end{bmatrix} \\ u_3 & \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \\ u_4 & \begin{bmatrix} 1 & 2 & 2 \end{bmatrix} \\ v_1 & \begin{bmatrix} 1 & 2 & 2 \end{bmatrix} \\ v_2 & \begin{bmatrix} 2 & 2 & 1 \end{bmatrix} \\ v_4 & \begin{bmatrix} 2 & 2 & 1 \end{bmatrix} \\ v_5 & \begin{bmatrix} 2 & 1 & 2 \end{bmatrix} \end{matrix}$$

Note that code of some pair of vertices are same i.e. $C_D(v_i) = C_D(v_j)$ for some $v_i, v_j \in V(G)$.

Therefore $D = \{u_1, u_5, v_3\}$ is not a resolving set of Petersen graph $G(V, E)$. Let $R = (D \cup \{v_{(i+5,3)}\})$

where $v_{(i+5,3)}$ is vertex on the outer pentagon. Let $R = \{u_1, u_5, v_3, v_5\}$, the code matrix of $C_R(G)$ is

$$C_R(G) = \begin{matrix} & u_1 & u_5 & v_3 & v_5 \\ u_2 & \begin{bmatrix} 2 & 1 & 2 & 2 \end{bmatrix} \\ u_3 & \begin{bmatrix} 1 & 1 & 1 & 2 \end{bmatrix} \\ u_4 & \begin{bmatrix} 1 & 2 & 2 & 2 \end{bmatrix} \\ v_1 & \begin{bmatrix} 1 & 2 & 2 & 1 \end{bmatrix} \\ v_2 & \begin{bmatrix} 2 & 2 & 1 & 2 \end{bmatrix} \\ v_4 & \begin{bmatrix} 2 & 2 & 1 & 1 \end{bmatrix} \end{matrix}$$

Note that in the code matrix the code of the of every pair of vertices are distinct i.e. $C_R(v_i) \neq C_R(v_j)$ for every $v_i, v_j \in V(G)$.

Therefore $R = \{u_i, u_{(i+5,4)}, v_{(i+5,2)}, v_{(i+5,3)}\}$ is the resolving set of Petersen graph $G(V, E)$ but not dominating set.

Since $D = \{u_i, u_{(i+5,4)}, v_{(i+5,2)}\}$ is a minimal dominating set of Petersen graph. Therefore R is a resolving but not dominating of Petersen graph $G(V, E)$. Hence $R = \{u_i, u_{(i+5,4)}, v_{(i+5,2)}, v_{(i+5,3)}\}$ is a minimal resolving set of Petersen graph.

The set R is a minimal resolving set or a basis of Petersen graph $G(V, E)$. Therefore dimension of Petersen graph $G(V, E)$ is cardinality of the basis R .

$$\begin{aligned} Dim(G) &= |R| \\ &= |\{u_i, u_{(i+5,4)}, v_{(i+5,2)}, v_{(i+5,3)}\}| \\ Dim(G) &= 4 \end{aligned}$$

Conclusion



Further we will work on different type of resolving set in graphs and also define the minimal resolving number of the different type of resolving set in graphs. In addition investigate the properties of that minimal resolving number of the different type of resolving set in graph.

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