



THERMAL RADIATION IMPACT ON MHD CASSON VISCO-ELASTIC FLUID UNDER VISCOUS DISSIPATION

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ABSTRACT

A theoretical study has been performed to analyze various properties of an electrically conducting visco-elastic and dissipative Casson fluid past a vertical porous plate bounded by a porous medium in the presence of thermal radiation and variable permeability. The basic concepts like magneto hydrodynamics, visco-elasticity, heat transfer, skin friction and rate of heat transfer are presented. The non-linear partial differential equations which govern the flow are solved numerically by finite difference scheme. The presence of thermal radiation decreases the temperature. The changes in skin friction and Nusselt number are also observed.

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Keywords: MHD, thermal radiation, Casson fluid, visco-elasticity, vertical porous plate, heat transfer.

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1. Introduction:

The studies on MHD visco-elastic fluids with radiation effect past a porous media plays significant role in many scientific, industrial and engineering applications. In MHD generator, the hot fluid is passing through transverse magnetic field, and then electric field will be produced. The liquid metal provides electrical conductivity and inert gas is a convenient carrier to the liquid. The carrier gas is pressurized and heated

by passage through the heat exchanger within the combustion chamber. The hot gas is incorporated into the liquid metal to form a working fluid. The liquid metal consists of gas bubbles uniformly dispersed in an approximately equal volume. The studies on MHD visco-elastic fluids with radiation effect past a porous media plays significant role in many scientific, industrial and engineering applications. To recover the water for drinking



and irrigation purposes the principles of this flow are followed.

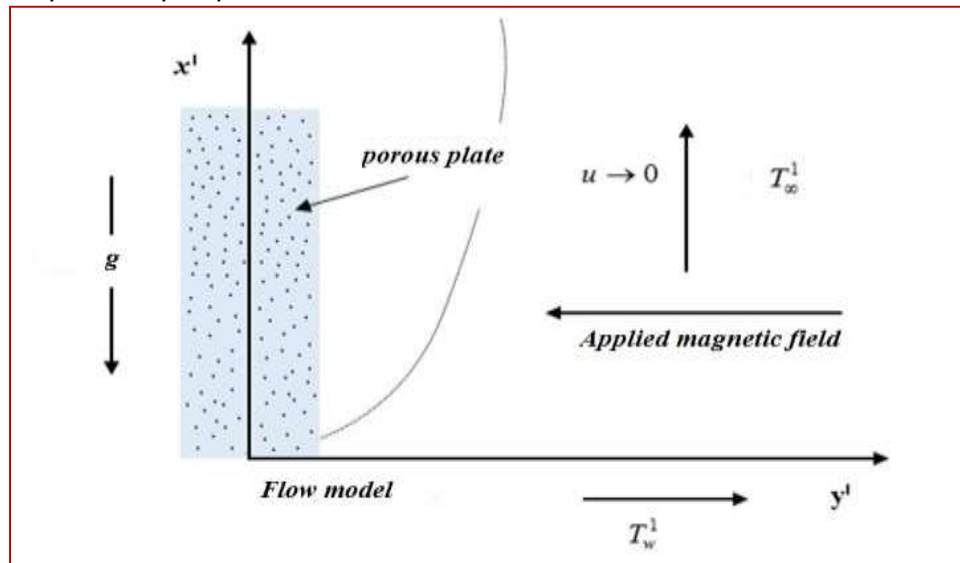
Many researchers identified the importance of this flows and contributed in studying the application of Casson visco-elastic fluid flow in the presence of thermal radiation. Rashidi et al. [1] established numerical investigation of magnetic field effect on mixed convection heat transfer of nanofluid in a channel with sinusoidal walls. Chandra Reddy et al. [2] analyzed magnetohydrodynamic convective double diffusive laminar boundary layer flow past an accelerated vertical plate. Mishra et al. [3] analyzed Mass and heat transfer effect on MHD flow of a visco-elastic fluid through porous medium with oscillatory suction and heat source. Harinath babu et al. [4] discussed MHD boundary layer flow of a visco-elastic and dissipative fluid the presence of thermal radiation. Chandra Reddy et al. [5] addressed buoyancy effects on MHD visco-elastic fluid past an inclined plate. Recently Hari Babu et al. [6,7,8] studied and analyzed the characteristics of non-Newtonian fluid flows with different geometries and variety of physical parameters. Chandra Reddy et al. [9] described analytical study of buoyancy effects on MHD

visco-elastic fluid past an inclined plate. Sivaiah et al. [10] studied numerically MHD boundary layer flow of a viscoelastic and dissipative fluid past a porous plate in the presence of thermal radiation. Recently Chandra Reddy et al. [11] studied numerically the parabolic flow of MHD fluid past a vertical plate in a porous medium.

Motivated by the above studies, in this article a viscous incompressible electrically conducting Casson fluid past a vertical porous plate bounded by a porous medium in the presence of thermal radiation, variable suction and variable permeability is analyzed.

2. Formulation of the problem:

The unsteady MHD Casson visco-elastic fluid flow past an infinite vertical porous plate with heat transfer embedded in a porous medium in the presence of thermal radiation, oscillatory suction as well as variable permeability is considered. A uniform magnetic field of strength B_0 is applied perpendicular to the plate. x^1 - axis is taken along with the plate in the direction of the flow and y^1 axis is normal to it.



Physical model of the fluid flow

Let us consider the magnetic Reynolds number is much less than unity so that the induced



magnetic field is neglected in comparison with the applied transverse magnetic field. The basic flow in the medium is, therefore, entirely due to the buoyancy force caused by the temperature difference between the wall and the medium. It is assumed that initially, at $t' \leq 0$, the plate as fluids are at the same temperature and

concentration. When $t' > 0$, the temperature of the plate is instantaneously raised to T'_w . Under the above assumption with usual Boussinesq's approximation, the governing equations and boundary conditions are given by (Mishra et al. [3], Harinath babu et al. [4]).

$$\frac{\partial u'}{\partial t'} + v \frac{\partial u'}{\partial y'} = v \frac{\partial^2 u'}{\partial y'^2} + g\beta(T' - T_\infty) - \frac{\sigma B_0^2 u'}{\rho} - \frac{vu'}{K'(t')} - \frac{k_0}{\rho} \left[\frac{\partial^3 u'}{\partial t' \partial y'^2} + v \frac{\partial^3 u'}{\partial y'^3} \right] \quad (1)$$

$$\frac{\partial T'}{\partial t'} + v \frac{\partial T'}{\partial y'} = K \frac{\partial^2 T'}{\partial y'^2} + S'(T' - T_\infty) + \mu \left(\frac{\partial u'}{\partial y'} \right)^2 - \frac{\partial q'_r}{\partial y'} \quad (2)$$

with the boundary conditions

$$u = 0, T' = T_w + \varepsilon(T_w - T_\infty)e^{n't'}, \quad \text{at } y' = 0 \quad (3)$$

$$u \rightarrow 0, T' \rightarrow T_\infty, \quad \text{as } y' \rightarrow \infty$$

The fluid considered here is optically thin with relatively low density, therefore the radiative heat flux is defined as $\frac{\partial q'_r}{\partial y} = 4(T' - T_\infty)I$. Let the permeability of the porous medium and the suction velocity be of the form

$$K'(t') = K'_p(1 + \varepsilon e^{n't'}) \quad (4)$$

$$v(t') = -v_0(1 + \varepsilon e^{n't'}) \quad (5)$$

where $v_0 > 0$ and $\varepsilon \ll 1$ are positive constants.

Introducing the non-dimensional quantities

$$y = \frac{v_0 y'}{v}, \quad t = \frac{v_0^2 t'}{4v}, \quad w = \frac{4vw'}{v_0^2}, \quad u = \frac{u'}{v_0}, \quad T = \frac{T' - T_\infty}{T_w - T_\infty},$$

$$S = \frac{vS'}{v_0^2}, \quad Kp = \frac{v_0^2 K'_p}{v^2}, \quad Pr = \frac{v}{K}, \quad M^2 = \frac{\sigma B_0^2 v}{\rho v_0^2},$$

$$Rc = \frac{k_0 v_0^2}{\sigma v^2}, \quad n = \frac{4vn'}{v_0^2}, \quad Gr = \frac{vg\beta(T_w - T_\infty)}{v_0^3}, \quad (6)$$

$$\frac{\partial q'_r}{\partial y} = 4(T' - T_\infty)I', \quad F = \frac{4vI'}{\rho C_p U_0^2}, \quad E = \frac{\mu U_0^2}{v \rho C_p (T_w - T_\infty)}$$



The equations (1), (2), (3) reduce to the following non-dimensional form:

$$\frac{1}{4} \frac{\partial u}{\partial t} - (1 + \varepsilon e^{nt}) \frac{\partial u}{\partial y} = \left(1 + \frac{1}{\gamma}\right) \frac{\partial^2 u}{\partial y^2} + GrT - M^2 u - \frac{u}{Kp(1 + \varepsilon e^{nt})} - \frac{Rc}{4} \frac{\partial^3 u}{\partial t \partial y^2} \quad (7)$$

$$\frac{1}{4} \frac{\partial T}{\partial t} - (1 + \varepsilon e^{nt}) \frac{\partial T}{\partial y} = \frac{1}{Pr} \frac{\partial^2 T}{\partial y^2} + E \left(\frac{\partial u}{\partial y}\right)^2 + ST - FT \quad (8)$$

with the boundary conditions: $u = 0, T = 1 + \varepsilon e^{nt}$, at $y = 0$

$$u \rightarrow 0, T = 0, \text{ as } y \rightarrow \infty \quad (9)$$

3. Method and Solution of the problem:

Equations (7)-(8) are coupled non-linear partial differential equations and are to be solved by using the initial and boundary conditions (9). The expressions for velocity u ($i, j+1$) and temperature T ($i, j+1$) are written and then $j+1^{\text{th}}$ level values are calculated by using j^{th} level values. The equivalent finite difference schemes of equations for (7)-(8) are as follows:

$$\left. \begin{aligned} \frac{1}{4} \frac{u_{i,j+1} - u_{i,j}}{\Delta t} - (1 + \varepsilon e^{nt}) \frac{u_{i+1,j} - u_{i,j}}{\Delta y} &= GrT_{i,j} + \left(1 + \frac{1}{\gamma}\right) \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{(\Delta y)^2} - M^2 u_{i,j} \\ - \frac{Rc}{4} \left(\frac{u_{i-1,j+1} - 2u_{i,j+1} + u_{i+1,j+1} - u_{i-1,j} + 2u_{i,j} - u_{i+1,j}}{\Delta t (\Delta y)^2} \right) &- \frac{1}{K_p (1 + \varepsilon e^{nt})} u_{i,j} \end{aligned} \right\} \quad (10)$$

$$\frac{1}{4} \left(\frac{T_{i,j+1} - T_{i,j}}{\Delta t} \right) - (1 + \varepsilon e^{nt}) \left(\frac{T_{i,j+1} - T_{i,j}}{\Delta y} \right) = \frac{1}{Pr} \left(\frac{T_{i-1,j} - 2T_{i,j} + T_{i+1,j}}{(\Delta y)^2} \right) + E \left(\frac{u_{i+1,j} - u_{i,j}}{\Delta y} \right)^2 + ST_{i,j} - FT_{i,j} \quad (11)$$

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Here, the index i refer to y and j to time. The mesh system is divided by taking $\Delta y = 0.1$. From equation (9), we have the following equivalent initial condition

$$u(i, 0) = 0, T(i, 0) = 0, \text{ for all } i$$

The boundary conditions from (9) are expressed in finite-difference form as follows

$$u(0, j) = 1, T(0, j) = 1, \text{ for all } j$$

$$u(i_{\max}, j) = 0, T(i_{\max}, j) = 0, \text{ for all } j$$

(Here i_{\max} was taken as 20). The procedure is repeated until $t = 0.5$ (i.e. $j = 500$). During computation Δt was chosen as 0.001.

Skin-friction: The skin-friction in non-dimensional form is given by the relation

$$\tau = - \left(\frac{du}{dy} \right)_{y=0}, \text{ where } \tau = \frac{\tau^1}{\rho U_0^2}$$



Nusselt number: The dimensionless rate of heat transfer in terms of Nusselt number is given by

$$Nu = -\left(\frac{d\theta}{dy}\right)_{y=0}$$

4. Results and observations:

The influence of physical parameters like Grashof number, magnetic parameter, thermal radiation, Prandtl number on velocity and temperature is discussed with the help of graphs. Figure 1 represents the changes in velocity under the influence of Casson fluid parameter. It is observed that velocity decreases when the values of Casson fluid parameter are increased. Figure 2 reveals that velocity decreases for raising values of magnetic field parameter. Thermal diffusion is the process of movement particles from one place to other place by temperature gradient. Figures 3 & 4 illustrates that the temperature decreases

under the effects of Prandtl number as well as thermal radiation.

Skin friction arises from the interaction between the fluid and the skin of the body, and is directly related to the wetted surface, the area of the surface of the body that is in contact with the fluid. Table 1 show that the skin friction coefficient reduces for increasing values of Eckert number. A reverse trend is shown in the case of radiation parameter and magnetic parameter. The rate of heat transfer increases under the influence of thermal radiation whereas it decreases in the case of Eckert number.

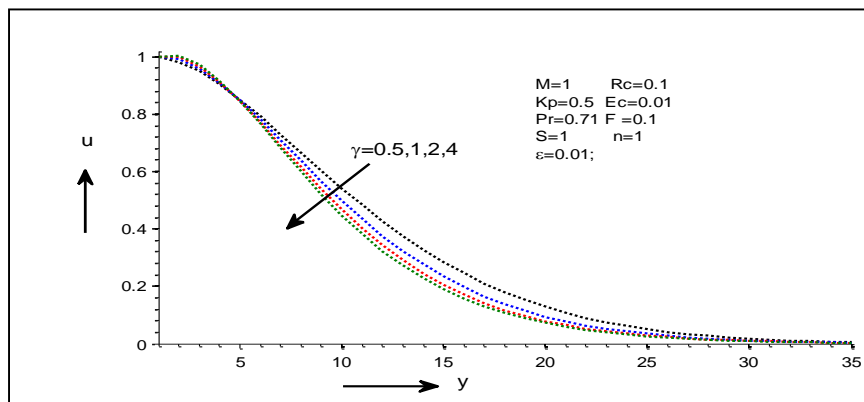


Figure 1: Velocity variations under the impact of Casson parameter



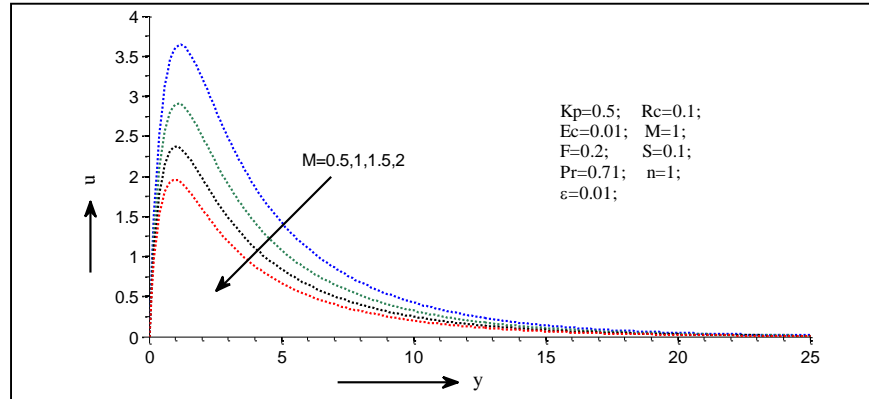


Figure 2: Velocity variation under the impact of magnetic parameter

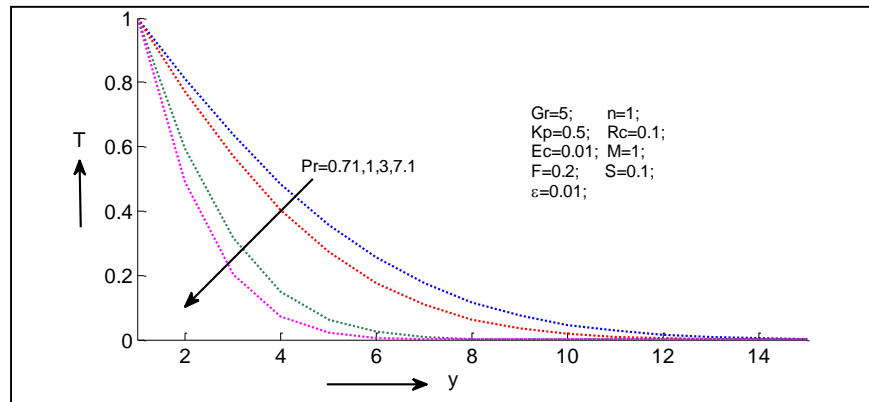


Figure 3: Temperature variations under the impact of Prandtl number

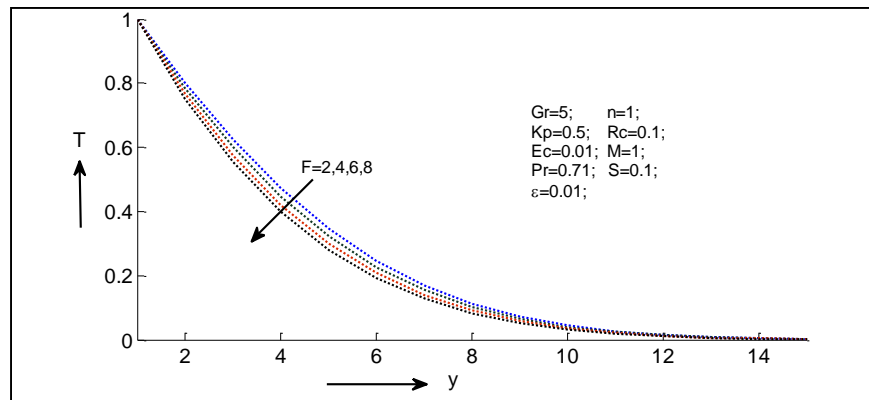


Figure 4: Temperature variations in the presence of thermal radiation

Table 1. Effect of various physical parameters on skin friction and Nusselt number



Ec	F	M	τ	Nu
0.01	1	5	2.058654	0.676623
0.03	1	5	1.978645	0.652245
0.05	1	5	1.539825	0.584414
0.26	2	5	2.034614	0.674478
0.26	4	5	2.085216	0.736285
0.26	6	5	2.157818	0.797948
0.26	1	1	6.968898	0.676265
0.26	1	1.4	7.966654	0.673448
0.26	1	1.8	9.322264	0.675492

Conclusion:

The governing equations for the velocity field, temperature and concentration by finite difference method. The main points observed from this study are as follows:

- Velocity decreases for increasing values of Casson fluid parameter and magnetic parameter.
- The temperature of the fluid decreases for rising values of Prandtl number and radiation parameter.
- Skin friction decreases with an increase of Eckert number and but a reverse effect is noticed in the case of magnetic parameter.
- Nusselt number increases as radiation parameter increases but in the case of Eckert number it decreases.

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