Paper – II Math

A Study New Integral Transform and Differential Equations

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DOI Number: 10.14704/nq.2022.20.8.NQ44001 **Neuro Quantology 2022; 20(8):9-13**

Integral transform are applied and helpful in the solution of partial differential equation. But, the choice of a particular transform to be used for the solution of a differential equation depends on the characteristic of the boundary condition of the equation. And the facility with which the transform f(p) can be inverted to provide –

$$f(s)=\int_{-\infty}^{\infty} e^{-st} d\alpha(t) \qquad (7.1,1)$$

Here, Let $\alpha(t)$ be of closely related variation in every finite interval. Particularly, if $\alpha(t)$ is an integral of a function $\phi(t)$, the integral (7.1, 1) becomes,

$$f(s)=\int_{-\infty}^{\infty} e^{-st}\phi(t)dt \qquad (7.1,2)$$

Laplace transforms or two sides Laplace transform by extending the limit of integration to be the entire real excess if that is done the common unilateral transform simply becomes a special case of integral transform

We say that $\alpha(t)$ is normalized in $(-\infty,\infty)$ if and only if $\alpha(0)=0$ and

$$\alpha(t)=\frac{\alpha(t+)+\alpha(t-)}{2} \qquad (-\infty<t<\infty). \qquad (7.1,3)$$

We observed a function normalized in interval $(0,\infty)$ and zero in $(-\infty,0)$ is not necessarily normalized in $(-\infty,\infty)$.

For the given values of s, whenever the integral (7.1,1) converges;

We found-

$$f(s)=\int_{0}^{\infty} e^{-st} d\alpha(t)+\int_{0}^{\infty} e^{st} d[-\alpha(-t)] \qquad (7.1,4)$$

So, the study of the bilateral transform is lowered to the total of two unilateral transforms in one of which the variable s has been substituted by –s.

Clearly, the bilateral transform is the continuous analogue of the Laurent series.

$$F(z) = \sum_{n=-\infty}^{\infty} a_n z^n, \qquad (7.2,15)$$

Where, f(z) be analytic in the ring shaped region surrounded by two concentric circles..

Axiom 7.2(a) If the following integral

$$f(s) = \int_{-\infty}^{\infty} e^{-st} d\alpha(t) \qquad (7.2,16)$$

Converges for two points $s_1 = \sigma_1 + i\tau_1$ others $s_2 = \sigma_2 + i\tau_2 (\sigma_1 < \sigma_2)$,

then, it is convergent in the vertical strip $\sigma_1 < \sigma < \sigma_2$.

As an outcome applied in a trivial manner, the integral first converges for $s = \sigma_0 + i\tau_0$ it converge for all $\sigma + i\tau$ for which $\sigma > \sigma_0$. For instance, the integral

$$\int_{-\infty}^{\infty} \frac{e^{-st}}{1+t^2} dt$$

converges actually on the whole line $\sigma = 0$ only. Lastly, the integral may have as its region of convergence certain parts of a vertical line. Thus if $\phi(t) = |t|^{-1/2}$, then the integral (7.2,15) converges on the line $\sigma = 0$ with an exception at the origin, and being divergent at all points off this line.

When the integral (7.2,16) is convergent and divergent in the strip $\sigma_c' < \sigma < \sigma_c''$ and $\sigma > \sigma_c''$ and for $\sigma < \sigma_c'$ then each of the lines $\sigma = \sigma_c'$ and $\sigma = \sigma_c''$ is known as an axis of convergence and each of the members σ_c' and σ_c'' is an abscissa of convergence.

Integration by parts

Here, we obtain an adequate & required conditions for the integration by parts of a Laplace integral. We first explain the two theorems regarding the nature of $\alpha(t)$ at $+\infty$ and at $-\infty$.

Axiom 7.3(a) If the integral

$$s\, f(s) = \int_{-\infty}^{\infty} e^{-st} d\alpha(t) \qquad (7.3,17)$$

converges for $s = s_0 = \gamma + i\delta$ with $\gamma > 0$ then $\alpha(-\infty)$ exists and

$$\alpha(t) = o(e^{\gamma t}) \qquad (t \to \infty)$$

$$\alpha(t)-\alpha(-\infty)=o\left(e^{\gamma t}\right) \qquad (t \to -\infty)$$

This result follows from the decomposition (7.3,17)

Axiom 7.3(b) If the integral (7.3,17) is convergent for $s = s_0 = \gamma + i\delta$ with $\gamma < 0$ then $\alpha(\infty)$ exists and

$$\alpha(t)-\alpha(\infty)=o\left(e^{\gamma t}\right) \qquad (t \to \infty)$$

$$\alpha(t)=o\left(e^{\gamma t}\right) \qquad (t \to -\infty)$$

This proof is related to the Axiom 3.3(a). But, above mentioned theorems seems to be failed if $\gamma = 0$

Axiom 7.3(c) If the integral (7.3,17) is convergent for $s = s_0 = \gamma + i\delta$ with $\gamma > 0$ and if $\alpha(-\infty) = 0,$ then we have integral of converges which $\sigma > 0$

$$f(s_0) = s_0 \int_{-\infty}^{\infty} e^{-s_0 t} \alpha(t) dt \qquad (7.3,18)$$

For,

$$\int_0^{\infty} e^{-s_0 t} d\alpha(t) = s_0 \int_0^{\infty} e^{-s_0 t} \alpha(t) dt - \alpha(0), \qquad (7.3,19)$$

converge with $\sigma < 0$ then $\alpha(\infty)$ exist

$$\int_0^{\infty} e^{s_0 t} d[-\alpha(-t)] = s_0 \int_0^{\infty} e^{s_0 t} \alpha(-t) dt + \alpha(0) \qquad (7.3,20)$$

On adding equations (7.3,19) & (7.3,20), we get equation (7.3,18).

Hence, a complete result !

Axiom 7.3(d) If the integral equation (7.3,17) is convergent for $s = s_0 = \gamma + i\delta$ with $\gamma < 0$ and if $\alpha(\infty) = 0,$ then

$$f(s_0) = s_0 \int_{-\infty}^{\infty} e^{-s_0 t} \alpha(t) dt\,.$$

The prior result by varying a variable t = - u. we have the restrictions following $\alpha(\infty) \text{and} \, \alpha(-\infty)$ in these two theorems are not serious ones, and these numbers exist by virtue of Axiom 3.3(a) and 3.3(b). Thus, the finishing of the one desired can always be brought about by the addition of a required constant to the determining function. As such, these addition has no effect on the generating function. We must not let (7.3,18) holds if $\gamma = 0$. Hence, if $\alpha(t)$ is the constant unity for positive and the constant zero for non-positive t, the above equation (7.3,17) exists for all value of s and has the value unity. But, if $s_0 = 0 + i$

*e*ISSN 1303-5150

$$s_0 \int_{-\infty}^{\infty} e^{-s_0 t} \alpha(t) dt = i \int_0^{\infty} e^{-it} dt,$$

a divergent integral

Conclusion:

This is such a partial differential equation whose solution finally contain some arbitrary constant and exponential function.

For the purpose of generating a new ideology and corollary on Non-Moment problem, we take into account equation (1) as the standard form of an equation representing it self as the generator of non-moment problem function. To make our result more accurate and widely considerable, we would like to introduce some other facts and finding coherent to the convergence of result and making our formula as a very common men discussion. In the Engineering Mechanics, ”Moment“ and in the basic function theory “exponential function” have been cause of concern for those who are working on Science and Technology. Though an exponential function generate an infinite series but consequently as far as number of terms in the series increases magnitude value of such particular term recedes. Like is the situation of impact of applied forces recedes as far as it touches molecular part of the physical substance in a Moment is produced. It is not always true to advovate that that this force plays its role in the one dimension part of the said and taken substance for the purpose of testing the Moment generated so. More than one variable are changed guiding the physical shape and molecular structure of the substance. Through out the study made learned Mathematician Pierre de Laplace (1749-1827), Engineer Oliver Heaviside (1850-1925),also by Bromwich and carson during 1916-17. Here we can some basic treatment on an exponential series that leads to the concept of a Non-Moment problem.

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