Theoretical Study of Photons Spectra around High Energy of Quark-antiquark Using QCD Theory

Elaf Mohammed Ahmed¹, Hadi J. M. Al-Agealy², Nada Farhan Kadhim³

Abstract

In this paper, we study and investigate the quark anti-quark interaction mechanism through the annihilation process. The production of photons in association with interaction quark and gluon in the annihilation process. We investigate the effect of critical temperature, strength coupling and photons energy in terms of the quantum chromodynamics model theory framework. We find that the use of large critical temperature $T_c = 134$ allows us to dramatically increase the strength coupling of quarks interaction. Its sensitivity to decreasing in photons rate with respect to strength coupling estimates. We also discuss the effect of photons energy on the rate of the photon, such as energies in range (1.5 to 5 GeV). The photons rate increases association at $T_c=116\text{MeV}$ with the more decreased photons energy compared with photons rate association at $T_c=116\text{MeV}$. This relation of strength coupling $a$, critical temperature and photons energy are particularly relevant when parametrizing systematic photons emission.

Key Words: Photons Spectra, High Energy, QCD Theory.

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Introduction

The elementary particle is a branch of physics is considered an unbreakable component of small particles and describes the basic building blocks of matter. The quarks idea was introduced in 1964 by both Zweig and Gill Mann independently as fundamental elements of hadrons[1]. The physical particles are considered of the most important field in physics. There are many scientific methods use to investigate and study many phenomena and facts to understanding and construction the nature. Various theories are introduced by scientists in different cite in world to study the interaction of quark-gluon and structure of nucleons [2]. The standard model was confirmed in 2012 when announcement the both CMS and Atlas research groups are detected Higgs boson. It was the final missing part of stander model. Standard model can be explained the collision process in variety research laboratories It ables to explain the structure of the nucleons up to now[3]. The fundamental particlesquarks and gluons are the constituents of the nucleons. The quarks and gluons play an important role to produce the mass of the nucleon and carry 1/2 of its momentum. The spin of nucleons are built up from the spin of quark is fermion has 1/2 and gluonis Boson has spin(1)[4]. The study of dynamics interaction quark and gluon that were understood due to QCD framework depending on heavy hadrons properties in regard understand the hadrons properties using different experiments for hadrons produces at the RHIC, BNL and CERN LHC[5]. Photons are produced in different collisions processes excluded the production of hadronic decays.

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The photons emit throughout the reaction of the heavy-ion medium, and they don’t experience subsequent interactions with the system because have color neutral [6]. There are different photons sources such that; prompt photons, thermal photons and photons produced in the pre-equilibrium by hard interactions. The photons create experimentally in proton-proton collisions and dominate the photons yield at high momentum in heavy ion collisions [7]. The quark-antiquark states are production in collisions, it has been investigated and discussed for evidence of transfer in nuclear matter, where gluons and quarks confine in hadronic matter [8]. In Figure, Feynman diagrams of photon produce mechanisms from, quark-antiquark annihilation.

![Feynman diagrams](image)

**Figure 1.** The diagrams of production the photon by quark-anti-quark annihilation [9].

In the last recently, there is a more interesting in strong react in QCD matter under high energy density condition. It is the quark-gluon plasma (QGP). It produces experimentally in collision of heavy ions at the Large Hadron Collider (LHC) in CERN or/and the relativistic collision at RHIC in Brook haven [10]. In addition, the photons emitted throughout the QGP and emission from the interaction medium are played an important role in that endeavor and unaffection by final state reaction [11]. This work will focus on the study and investigate to produce of photons from interaction of quark and anti-quark in annihilation processes.

**Theory**

The photons spectrum from quark anti quarks annihilation is given by [12]

\[
R_\gamma = \frac{4n_s^2}{(2\pi)^2} F_q(p_\gamma) \int F_g(p_\gamma) [1 + F_g(p_\gamma)\sigma_{qq}(s) \frac{s^2}{2E_\gamma}] d^3p_q (1)
\]

Where \(n_s\) is number of quark spins, \(F_q(p_\gamma)\) and \(F_g(p_\gamma)\) are Juttner distribution for quark and gluon, \(d^3p_q\) is element volume interaction and \(\sigma_{qq}(s)\) (s is total cross section).

The element volume \(d^3p_q\) is written as [13].

\[
d^3p_q \approx \frac{E_\gamma}{2E} d\sigma d\phi dE_q (2)
\]

Where \(E_q\) and \(E_\gamma\) are quark and photons energy, \(d\sigma, d\phi\) and \(dE_q\) are element of momentum, solid angle and quarks energies.

The cross section of annihilation process \(\sigma_{qq}(s)\) is [14].

\[
\sigma_{qq}(s) = \left(\frac{\alpha}{\pi}\right)^2 \sigma_a(s) (3)
\]

Where \(\sigma_a\) and \(e\) are the quarks and electronic charges and \(\sigma_a(s)\) is the effective cross section.

Inserting Eq.(2) and (3) in Eq.(1) to obtained

\[
R_\gamma = \frac{4n_s^2}{(2\pi)^2} F_q(p_\gamma) \left(\frac{E_\gamma}{\pi}\right)^2 \frac{1}{4E_\gamma} \int F_g(p_\gamma) [1 + F_g(p_\gamma)\sigma_a(s)\sqrt{s(s - 4m^2)} ds \int_0^{2\pi} d\phi] (4)
\]

The distribution of quark according Juttner function is given by [15].

\[
F_q(E_\gamma) = \frac{\lambda_q}{e^{\frac{E_\gamma}{\mu_q}} - 1} (5)
\]

Where \(\lambda_q\) is fugacity of anti-quark. Bose-Einstein distribution \(F_B(E_\gamma)\) for gluon is [16].

\[
F_g(E_\gamma) = \frac{\lambda_g}{e^{\frac{E_\gamma}{\mu_g}} - 1} (6)
\]

Inserting Eqs.(5) and (6) in Eq.(4) and carry out the first integral over \(E_\gamma\) under the condition taken \(E_\gamma \geq \frac{s}{4E_\gamma}\) [17].

\[
R_\gamma = \frac{4n_s^2}{(2\pi)^2} F_q(p_\gamma) \left(\frac{E_\gamma}{\pi}\right)^2 \frac{1}{4E_\gamma} \int \sigma_a(s)\sqrt{s(s - 4m^2)} ds \int_0^{2\pi} d\phi (7)
\]

The solution of the first integral for case \(E_\gamma \equiv E_q\) is

\[
\int_0^{\infty} \frac{\lambda_q}{e^{\frac{E_\gamma}{\mu_q}} + \lambda_g e^{\frac{E_\gamma}{\mu_g}}} dE_q = \int_0^{\infty} \left[\frac{\lambda_q}{\mu_q} + \frac{\lambda_g}{\mu_g} \frac{1}{e^{\frac{E_\gamma}{\mu_g}} + 1} \right] dE_q = T\lambda_q \Sigma_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \left(\frac{\lambda_g}{\mu_g} + \frac{\lambda_q}{\mu_q}\right) e^{-\frac{E_\gamma}{\mu_g}} (8)
\]

And the third integral is

\[
\int_0^{2\pi} d\phi = 2\pi (9)
\]

We can insert Eq.(8) and Eq.(9) in Eq.(7), results

\[
R_\gamma = \frac{4n_s^2}{(2\pi)^2} F_q(p_\gamma) \left(\frac{E_\gamma}{\pi}\right)^2 \int [T\lambda_q \Sigma_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \left(\frac{\lambda_g}{\mu_g} + \frac{\lambda_q}{\mu_q}\right) e^{-\frac{E_\gamma}{\mu_g}}] + \int_0^{\infty} \lambda_q \lambda_g \Sigma_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \left(\frac{\lambda_g}{\mu_g} + \frac{\lambda_q}{\mu_q}\right) e^{-\frac{E_\gamma}{\mu_g}} dE_q (10)
\]

But [13].

\[
\int_0^{\infty} \sqrt{s(s - 4m^2)} \sigma(s) ds = 4\pi\alpha_\sigma s^2 \int \frac{\ln\left(\frac{m^2}{s^2}\right)}{s^2} ds (11)
\]

Inserting Eq.(11) in Eq.(10) with \(s > 4m^2\) to reduced

\[
R_\gamma = \frac{4n_s^2}{(2\pi)^2} F_q(p_\gamma) \left(\frac{E_\gamma}{\pi}\right)^2 4\pi\alpha_\sigma m^2 T_\lambda q [\Sigma_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \left(\frac{\lambda_g}{\mu_g} + \frac{\lambda_q}{\mu_q}\right) e^{-\frac{E_\gamma}{\mu_g}}] + \lambda_q \lambda_g \Sigma_{n=1}^{\infty} \frac{1}{n} \left(\frac{\lambda_g}{\mu_g} + \frac{\lambda_q}{\mu_q}\right) e^{-\frac{E_\gamma}{\mu_g}} \ln\left(\frac{m^2}{s^2}\right) - 1 ds (12)
\]
we will assume \( s = 4m^2z \) to obtained

\[
R_y = \frac{4\alpha_T}{(2\pi)^2} \left( \frac{e^2}{\pi} \right) \frac{1}{4} 4\alpha_0 a_T m^2 \gamma \sum_{n=1}^{\infty} \left( \frac{-1}{n} \right)^{n+1} \left( \frac{E_T}{m^2} \right)^{n+1} \ln \left( \frac{E_T}{m^2} \right) - \frac{C - lnn - \ln(2n+1) - 1}{n^2} \]

And

\[
\sum_{n=1}^{\infty} \left( \frac{-1}{n} \right)^{n+1} - \frac{\pi^2}{6} \]

Subsume Eq.(14) and Eq.(15) in Eq.(13) to results

\[
R_y = \frac{4\alpha_T}{(2\pi)^2} \left( \frac{e^2}{\pi} \right) \frac{1}{4} 4\alpha_0 a_T m^2 \gamma \sum_{n=1}^{\infty} \left( \frac{-1}{n} \right)^{n+1} \left( \frac{E_T}{m^2} \right)^{n+1} \ln \left( \frac{E_T}{m^2} \right) - \frac{C - lnn - \ln(2n+1) - 1}{n^2} \]

But the power series reduced to [18].

\[
\sum_{n=1}^{\infty} \left( \frac{-1}{n} \right)^{n+1} = 0 \]

And

\[
\sum_{n=1}^{\infty} \left( \frac{-1}{n} \right)^{n+1} - \frac{\pi^2}{6} \]

we get

\[
R_y = \frac{4\alpha_T}{(2\pi)^2} \left( \frac{e^2}{\pi} \right) \frac{1}{4} 4\alpha_0 a_T m^2 \gamma \sum_{n=1}^{\infty} \left( \frac{-1}{n} \right)^{n+1} \left( \frac{E_T}{m^2} \right)^{n+1} \ln \left( \frac{E_T}{m^2} \right) - \frac{C - lnn - \ln(2n+1) - 1}{n^2} \]

The Juttner distribution function for quark in Eq.(5) for \( E_y \gg T \) reduce to

\[
\frac{\lambda_q}{e^{T+\frac{E_y}{T^+1}}} \]

Then inserting Eq.(20) in Eq.(19) and assume the annihilation coefficient \( C_{an} = C + 1 + lnn + \ln(2n+1) \) with \( n_s = 2 \) to obtain.

\[
R_y = \frac{\alpha_0 a_T}{3(2\pi)^2} \left( \frac{e^2}{\pi} \right) \frac{1}{4} 4\alpha_0 a_T m^2 \gamma \sum_{n=1}^{\infty} \left( \frac{-1}{n} \right)^{n+1} \left( \frac{E_T}{m^2} \right)^{n+1} \ln \left( \frac{E_T}{m^2} \right) - C_{an} \]

Where \( \alpha_0 \) is the electrodynamic strength constant

\[
\alpha_0 = \frac{e^2}{\pi \epsilon_0} = (137)^{-1} \]

\( \alpha_s \) is the chromodynamic strength constant, \( \lambda_q \) and \( \lambda_q^* \) are the fugacity of quarks and anti quarks with finite quark mass \( m \) is written as[19]

\[
m = gT = \sqrt{4\alpha_s T} \]

The Eq.(21) with Eq.(22) reduced to

\[
E_y \frac{d\gamma}{d^2\gamma} = \frac{\alpha_0 a_T}{3(2\pi)^2} \left( \frac{e^2}{\pi} \right) \frac{1}{4} 4\alpha_0 a_T m^2 \gamma \sum_{n=1}^{\infty} \left( \frac{-1}{n} \right)^{n+1} \left( \frac{E_T}{m^2} \right)^{n+1} \ln \left( \frac{E_T}{m^2} \right) - C_{an} \]

The quantum chromo dynamic strength coupling is calculated using [20].

\[
\alpha_s = \frac{6\pi}{(33-2n_f)\ln^{\gamma}} \]

The transition energy is [21].

\[
T_c = \frac{90\theta}{\pi^3 n_{gg}} \]

Where \( B \) is the Bag constant with bag model and \( n_{gg} \) is the number of gluons and quarks degrees of freedom. It is given by [22].

\[
n_{gg} = n_g + \frac{2}{6} (n_q + n\bar{q}) \]

**Results**

The photons spectra that are emission from interaction quark anti-quark constituent of QGP calculation is calculated depending on the distribution function for state of the fermions (quark and anti-quark) and gluon (boson) at critical temperatures (110 and 126 MeV). To calculate the rate of the photon, we must estimation the quarks charge and flavour number of system, the quarks charge is obtained through the summation charge

\[
\sum \left( \frac{e^2}{\pi} \right) = \frac{2}{9} \]

system where charge of strange is \( e_q = -1/3 \) and anti-strange is \( e_{\bar{q}} = 1/3 \). While the net favor number is \( n_f = 8 \) for \( s\bar{s} \rightarrow yg \) interaction system. To evaluate the critical temperature from Eq.(25), we estimate the degrees of freedom of the quark-gluon system from Eq.(26) by taking the \( n_s = n_g = n_c = 16 \) where \( n_s = 2 \) and \( n_c = 8 \) are the gluons spin and color states and \( n_q = n_{\bar{q}} = n_c \times n_s \times n_f = 48 \) while \( n_c = 3 \), \( n_s = 2 \) and \( n_f = 8 \) are the number of quark colour, spin and flavour degrees of freedom to results \( n_{gg} = 58 \). We construct the critical temperature of quarks and gluons system by using Eq.(25) with taking the Bag constant \( B^{1/4} = 200 \) and 230 MeV [23] to, the results of critical estimation are shown in table(1).

<table>
<thead>
<tr>
<th>Bag constant ( B^{1/4} ) MeV</th>
<th>Critical temperature ( T_c ) MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>116</td>
</tr>
<tr>
<td>230</td>
<td>134</td>
</tr>
</tbody>
</table>

*Table 1. Result of critical temperature calculation using the Bag mode of the quarks system for \( s\bar{s} \rightarrow yg \) System*
Additionally, the strength coupling contributions can be calculation using Eq.(24) with the take the critical temperature from table (1) and flavor number \( n_f = 6 \) for \( s\bar{s} \to \gamma g \) system, results shows in table (2).

<table>
<thead>
<tr>
<th>( T \text{MeV} )</th>
<th>The strength coupling ( \alpha_s ) at ( T_c = 116 \text{ MeV} )</th>
<th>The strength coupling ( \alpha_s ) at ( T_c = 134 \text{ MeV} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>170</td>
<td>0.36463</td>
<td>0.38732</td>
</tr>
<tr>
<td>190</td>
<td>0.34886</td>
<td>0.36959</td>
</tr>
<tr>
<td>210</td>
<td>0.33580</td>
<td>0.35496</td>
</tr>
<tr>
<td>230</td>
<td>0.32475</td>
<td>0.34263</td>
</tr>
<tr>
<td>250</td>
<td>0.31524</td>
<td>0.33206</td>
</tr>
<tr>
<td>270</td>
<td>0.30694</td>
<td>0.32287</td>
</tr>
</tbody>
</table>

In addition to strength coupling in table(2), we take the photon energy \( E_{\gamma} = 1.5 \) to 5GeV, \( T=170 \) to 270 MeV and fugacity of quark and anti-quark are \( \lambda_q = 0.06, \lambda_{\bar{q}} = 0.06 \) respectively with taking Eq.(23) to find the photons spectrum rate where the annihilation coefficient is \( C_{an} = 1.415 \text{[24]} \). Results are shown in table (3) and table (4) and figures (1) and (2) respectively for critical temperature \( T_c = 116 \) and 134 MeV.

<table>
<thead>
<tr>
<th>( E_{\gamma} ) GeV</th>
<th>( \Gamma_{qs}(E_{\gamma}, p) ) ( \frac{1}{\text{GeV}^2 f m^4} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>4.7928 5E-14 2.9272 2E-14 1.2949 1E-14 0.3644 0E-14 1E-09 0.5480 8E-12 9.3054 1E-11 1.4337 1E-12</td>
</tr>
<tr>
<td>2</td>
<td>3.6927 9E-15 2.2595 5E-14 1.2868 1E-12 1.4110 1E-10 1.2949 2E-10 0.3644 0E-12 1E-09 0.5480 8E-11 9.3054 1E-11 1.4337 1E-12</td>
</tr>
<tr>
<td>2.5</td>
<td>3.6927 9E-15 2.2595 5E-14 1.2868 1E-12 1.4110 1E-10 1.2949 2E-10 0.3644 0E-12 1E-09 0.5480 8E-11 9.3054 1E-11 1.4337 1E-12</td>
</tr>
<tr>
<td>3</td>
<td>2.5672 9E-15 2.2595 5E-14 1.2868 1E-12 1.4110 1E-10 1.2949 2E-10 0.3644 0E-12 1E-09 0.5480 8E-11 9.3054 1E-11 1.4337 1E-12</td>
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</tr>
</tbody>
</table>
Discussion

Depending on results, we presented calculated results for the calculation of critical temperature, strength coupling and photons rate produced and also the photons spectrum from the interaction of quarks through the annihilation process at finite temperature (170 - 270 MeV) with using fugacity of quark and anti-quark. The evaluation has been done in two different critical temperature and quarks number flavor $n_f = 6$. Photons spectrum is plotted in Figures 1 and 2. In Figures 1 and 2, the rate of photons are produced with different initial temperatures for variety quark flavors in annihilation processes. The photons rate is to be increased due to increase temperature of the system and decreased the strength coupling in both critical temperature of quark flavor $n_f = 6$. In Figure 1, the photons spectrum with the critical temperature 110MeV is less compare with the photons spectrum in Figures 2 with the critical temperature 126MeV. It indicates that the use of flavour number, fugacity, freedom number and photons energy parameters in the photons rate in two figures enhancement the photons rate calculation and creation the stability in the 1.5 GeV of the photons rate evolution. The calculation at such stable photons rate indicate that producing the rate of photon in the energy range 1.5–2.0 GeV. Then the increased of temperature of system $T > 170$ MeV indicate the photons produced increased with instable rate and the photons rate would be more with the increased temperature. However, we can believe it performing the photon produces for unstable interaction with suitable choices of annihilation processes parameters at the photons energies $E_\gamma \geq 2$GeV. It can be shown a clear cut about the stabilities of photons emission for the different temperature at the photons energy $E_\gamma \geq 2$GeV for both figures. Similarly, we can find the photons produced increases with decreases the strength coupling in table (3) and (4). The photons produced are found to be large at large critical temperature $T_c = 134$ MeV compared to the photons produced at low critical temperature $T_c = 116$ MeV with the same systems $\bar{s} \rightarrow \gamma g$ in interaction system form with the quark number flavor $n_f = 6$ at similar temperature $T = 170$ to 270 MeV. The emission photon rate is increasing in the highly and to effect by the increased temperature. It shows to be large near temperature system $T = 270$ MeV. On the other hand, the suppression in the photon emission is less for both figures with increase photon energy at $T = 0.270$ GeV. Because of that suppression of emission rate increases for increases the photon energy $E_\gamma \gg T$. From both Figures 1 and 2, we can see the comparing results of photons rate produces at different temperature $T = 0.170$ to 0.270n GeV for annihilation process of the photons produce. We can find that the photons emission rate contributions effective higher in lowering photon energy $E_\gamma = 1.5$ GeV in compared to large photons energy for annihilation processes, and it is less dominated in the higher energy for both critical temperature.

Conclusion

We conclude that the strength coupling and critical temperature are influenced largely on the photon emission yield of the annihilation interaction through the number of freedom and flavor number parameterization factors enhancement the stability of photons emission. Results calculation of emission photon produce and photons rate is a function of photon energy and strength coupling, there are incorporated to give improve data result in emission photon spectrum. Thus, the consideration of critical temperature and temperature parameters have an important role in emission of the photon in the higher energy collisions.

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