

# On the Physical Basis of Theory of “Mental Waves”

Andrei Khrennikov

## Abstract

We discuss the conjecture on quantum-like (QL) processing of information in the brain. It is not based on the physical quantum brain (*e.g.*, Penrose) - quantum physical carriers of information. In our approach the brain created the QL representation (QLR) of information in Hilbert space. It uses quantum information rules in decision making. The existence of such QLR was (at least preliminary) confirmed by experimental data from cognitive psychology; in particular, experiments on recognition of ambiguous figures (Conte *et al.*, 2006; 2009). The violation of the law of total probability in these experiments is an important sign of nonclassicality of data. In this paper we speculate on a possible physical realization of QLR in the brain: a classical wave model producing QLR. It is based on variety of time scales in the brain. Each pair of scales (fine - the background fluctuations of electromagnetic field and rough - the cognitive image scale) induces the QL representation. The background field plays the crucial role in creation of “superstrong QL correlations” in the brain.

**Key Words:** quantum-like model, information processing, brain, Hilbert space representation, classical wave model, background field, electromagnetic field in brain

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## 1. Introduction

Many authors (*e.g.*, Roger Penrose and Stuart Homeroff) advertized the model of the quantum brain, *i.e.*, quantumness of the brain is a consequence of its composition of quantum systems. Of course, such a brain is a processor of quantum information. This approach induces numerous complicated questions on physics of such brain, *e.g.*, “Is the brain too hot to be quantum?” Therefore,

although we do not deny completely this interesting model, we do not couple our mathematical model of quantum information processing in the brain to the “*physical quantum brain*.”

In 2001 I pointed (Khrennikov, 2010) to coupling between violation of the law of total probability (LTP) and interference of probabilities in quantum mechanics, *e.g.*, in the fundamental two slits experiment. Interference (both classical and quantum) implies the violation of LTP; moreover, violation of LTP induces the wave-like representation of probabilistic data by complex (and more general) amplitudes - the *constructive wave function approach* (Khrennikov, 2009a). LTP plays the

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Corresponding author: Andrei Khrennikov

Address: International Center for Mathematical Modeling in Physics and Cognitive Sciences Linnaeus University, S-35195, Växjö, Sweden. Institute of Information Security Russian State University for Humanities, Moscow, Russia.

e-mail: andrei.khrennikov@lnu.se

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fundamental role in decision making; its violation implies a new strategy in decision making - nonclassical decision making (Khrennikov, 2006a).

We point out that LTP is violated in some experiments of cognitive psychology, e.g., experiments on recognition of ambiguous figures (Conte *et al.*, 2006; 2009) or games of the Prisoner's Dilemma (PD) type (Khrennikov, 2004; 2010). The violation of LTP in these experiments is an important sign of nonclassicality of cognitive data. In the constructive wave function approach that such data can be represented by complex probability amplitudes (Khrennikov, 2001; 2004). This is an important motivation to look for QL models of information processing in the brain.

In previous works (Khrennikov, 2010) we presented quantum information models of decision making in games of the PD-type. This is a model of how the brain using QLR of information might work. Thus, on one hand, we have experimental data which support the hypothesis of QL processing of information in the brain and, on the other hand, we have a theoretical model of such processing.

In this paper I am looking for models of physical realization of QLR in the brain. We propose a classical (!) wave model which reproduces probabilistic effects of quantum information theory. Why do we appeal to classical electromagnetic fields in the brain and not to quantum phenomena? In neurophysiological and cognitive studies we see numerous classical electromagnetic waves in the brain. Our conjecture is that these waves are carriers of mental information which is processed in the framework of quantum information theory.

In the quantum community there is a general opinion that quantum effects can not be described by classical wave models (however, *cf.* Schrödinger). Even those who agree that the classical and quantum interferences are similar emphasize the role of quantum entanglement and its irreducibility to classical correlations (however, *cf.* Einstein-Podolsky-Rosen). It is well known that entanglement is crucial in quantum information theory. Although some authors emphasize the role of quantum parallelism in quantum computing, i.e.,

superposition and interference, experts know well that without entanglement the quantum computer is not able to beat the classical digital computer.

Recently I propose a classical wave model reproducing all probabilistic predictions of quantum mechanics, including correlations of entangled systems, so called *prequantum classical statistical field theory* (PCSFT), (Khrennikov, 2005a; 2005b; 2006a-d), and see paper (Khrennikov, 2009c) for the recent model for composite systems. It seems that, in spite of mentioned common opinion, the classical wave description of quantum phenomena is still possible.

In this paper we apply PCSFT to model QL processing of information in the brain on the basis of classical electromagnetic fields. This model is based on the presence of various *time scales in the brain*. Roughly speaking each pair of time scales, one of them is fine - the background fluctuations of electromagnetic (classical) field in the brain, and another is rough - the cognitive image scale, can be used for creation of QLR in the brain. The background field (background rhythms in the brain) which is an important part of our model plays the crucial role in the creation of "superstrong QL correlations" in the brain. These mental correlations are nonlocal due to the background field. These correlations might provide a solution of the *binding problem*.

Each such a pair of time scales, (fine, rough), induces QLR of information. As a consequence of variety of time-scales in the brain, we get a variety of QL representations serving for various psychological functions. This QL model of brain's work was originated in author's papers (Khrennikov, 2008b; 2009b). The main improvement of the "old model" is due to a new possibility achieved recently by PCSFT: to represent the quantum correlations for entangled systems as the correlations of the classical random field, so to say prequantum field. This recent development also enlightened the role of the background field, vacuum fluctuations. We now transfer this mathematical construction designed for quantum physics to the brain science. Of course, it is a little bit naive model, since we do not know the "QL code" used by the brain: the correspondence

between images and probability distributions of random electromagnetic fields in the brain.

## 2 The role of the law of total probability

### 2.1 LTP and classical decision making

We recall the classical LTP: *The prior probability to obtain the result, e.g.,  $b=+1$  for the random variable  $b$  is equal to the prior expected value of the posterior probability of  $b=+1$  under conditions  $a=+1$  and  $a=-1$ ;*

$$P(b=j) = P(a=+1)P(b=j|a=+1) + P(a=-1)P(b=j|a=-1), \quad (1)$$

where  $j=+1$  or  $j=-1$ .

LTP gives a possibility to predict the probabilities for the  $b$ -variable on the basis of conditional probabilities and the  $a$ -probabilities. The main idea behind applications of LTP is to split in general complex condition, say  $C$ , preceding the decision making for the  $b$ -variable into a family of (disjoint) conditions, in our case  $C_+^a = \{a=+1\}$  and  $C_-^a = \{a=-1\}$ , which are less complex. Then one estimate in some way (subjectively or on the basis of available statistical data) the probabilities under these simple conditions:  $P(b=\pm 1|a=\pm 1)$  and the probabilities  $P(a=\pm 1)$  of realization of conditions  $C_\pm^a$ . On the basis of these data LTP provides the value of the probability  $P(b=j)$  for  $j=\pm 1$ . If, e.g.,  $P(b=+1)$  is larger than  $P(b=-1)$ , it is reasonable to make the decision  $b=+1$ , say yes.

Typically decision making is based on two thresholds for probabilities (assigned depending a problem):  $0 \leq \varepsilon_- \leq \varepsilon_+ \leq 1$ . If the probability  $P(b=+1) \geq \varepsilon_+$ , the decision  $b=+1$  should be done. If the probability  $P(b=+1) \leq \varepsilon_-$ , i.e.,  $P(b=-1) \geq 1 - \varepsilon_-$ , the decision  $b=-1$  should be done. If  $\varepsilon_- < P(b=+1) < \varepsilon_+$ , then an additional analysis should be performed.

My basic conjecture was that *cognitive systems developed the ability to use nonclassical LTP for decision making:*

$$P(b=\pm 1|C) = P(a=+1|C)P(b=\pm 1|a=+1) + P(a=-1|C)P(b=\pm 1|a=-1) + 2 \cos \theta_\pm \sqrt{\Pi_\pm}$$

where

$$\Pi_\pm = P(a=+1|C)P(b=\pm 1|a=+1) \times P(a=-1|C)P(b=\pm 1|a=-1)$$

This formula (the classical LTP perturbed by so called interference term) can be easily derived in the formalism of quantum mechanics where observables  $a$  and  $b$  are represented by self-adjoint operators (Khrennikov, 1999). We can derive (Khrennikov, 2009a) it without appealing to the Hilbert space formalism, namely, by controlling contextual dependence of probabilities. We recall that mathematically *contextuality of probabilities* is equivalent to non-Kolmogorovness of probabilistic data.

### 2.2 Violation of LTP from contextuality of probabilities

In particular, LTP is violated in quantum physics, in the two slit experiment. The  $b$ -observable gives the position of photon on the registration screen. If one likes to couple coming considerations to the decision making, she can consider the problem of prediction of the position of photon's registration: to predict the probability that photon hits a selected domain on the registration screen.

To make the  $b$ -variable discrete, we split the registration screen into two domains say  $B_+$  and  $B_-$  and if a photon makes the black dot in  $B_+$ , we set  $b=+1$ , and in the same way we define the result  $b=-1$ .

The  $a$ -variable describes the slit which is used by a particle; say  $a=+1$  the upper slit and  $a=-1$  the lower slit. For simplicity, we set  $P(a=+1) = P(a=-1) = 1/2$ , so the source is placed symmetrically with respect to slits. Consider three different experimental contexts:

$C$ : both slits are open. We can find  $P(b=+1)$  and  $P(b=-1)$  from the experiment as the frequencies of photons hitting the domains  $B_+$  and  $B_-$ , respectively.

$C_+^a$ : only one slit, labeled by  $a=+1$ , is open. We can find  $P(b=j|a=+1)$ ,  $j=\pm 1$ , the frequencies of photon hitting  $B_+$  and  $B_-$ , respectively.

$C_-^a$ : only one slit, labelled by  $a = -1$ , is open. We can find  $P(b = j | a = -1), j = \pm 1$ , the frequencies of photon hitting  $B_+$  and  $B_-$ , respectively.

If we put these frequency-probabilities, collected in the three real experiment, we see that LTP is violated. The classical LTP cannot be used to predict, e.g., the probability  $P(b = +1)$  that photon hits  $B_+$  under the context  $C$  (both slits are open) on the basis of probabilities  $P(b = j | a = \pm 1)$ , that photon hits  $B_+$  under contexts  $C_{\pm}^a$  (only one respective slit is open).

### 2.3 Violation of LTP in cognitive science

Data obtained in experiments of cognitive psychology, Tversky-Shafir, Croson, see book of Khrennikov (Khrennikov, 2010) for details, demonstrated violation of LTP. As in the above analysis of the two slit experiment, incompatible contextual structures can be easily found in all these cognitive experiments. We emphasize that here violation of LTP is even more general than described by the Dirac-von Neumann formalism of the standard quantum mechanics (as, e.g., in the two slit experiment). Thus processing of information in cognitive systems is even more nonclassical than in quantum physics. One of possibilities to proceed is to use quantum Markov chains, see Accardi, Khrennikov and Ohya (Accardi *et al.*, 2009): a concrete quantum Markov chain reproducing data from Tversky-Shafir experiment was constructed. We also mention quantum-like interference experiments on recognition of ambiguous figures (Conte *et al.*, 2006; 2008, Khrennikov, 2010), which were designed on the basis of the author's paper (Khrennikov, 2004). It seems that their results can neither be reproduced on the basis of quantum Markov chains.

Recently Asano, Khrennikov and Ohya (Asano *et al.*, Unpublished, In Press) proposed a generalized quantum model based on so called liftings of density operators modeling the process of decision making in the PD-type games.

### 2.4 Wave representation of information in the brain?

One may come with the conjecture that decision making with nonclassical LTP is based on a kind of the wave representation of information in the brain. The brain is full of classical electromagnetic radiation. May be the brain was able to create QLR of information via classical electromagnetic signals, cf. K.-H. Fichtner, L. Fichtner, W. Freudenberg and M. Ohya (Fichtner *et al.*, 2008).

Classical waves produce superposition and violate LTP. However, quantum information processing is based not only on superposition, but also on ENTANGLEMENT. It is the source of superstrong nonlocal correlations. Correlations are really superstrong - violation of Bell's inequality. Can entanglement be produced by classical signals? Can quantum information processing be reproduced by using classical waves? The answer is positive.

The crucial element of coming wave model is the presence of the random background field (in physics fluctuations of vacuum, in the cognitive model - background fluctuations of the brain). Such a random background increases essentially correlations between different mental functions, generates nonlocal presentation of information. We might couple these nonlocal representation of information to the binding problem:

*"How the unity of conscious perception is brought about by the distributed activities of the central nervous system."*

### 3. Prequantum classical statistical field theory: noncomposite systems

Quantum mechanics (QM) is a statistical theory. It cannot tell us anything about an individual quantum system, e.g., electron or photon. It predicts only probabilities for results of measurements for ensembles of quantum systems. Classical statistical mechanics (CSM) does the same. Why are QM and CSM based on different probability models?

In CSM averages are given by integrals with respect to probability

measures and in QM by traces. In CSM we have:

$$\langle f \rangle_\mu = \int_M f(\phi) d\mu(\phi),$$

where  $M$  is the state space. In probabilistic terms: there is given a random vector  $\phi(\omega)$  taking values in  $M$ . Then  $\langle f \rangle_\phi = Ef(\phi(\omega)) = \langle f \rangle_\mu$ . In QM the average is given by the operator trace-formula:

$$\langle \hat{A} \rangle_\rho = \text{Tr} \rho \hat{A}.$$

This formal mathematical difference induces the prejudice on fundamental difference between classical and quantum worlds. Our aim is to show that, in spite of the common opinion, quantum averages can be easily represented as classical averages and, moreover, even correlations between entangled systems can be expressed as classical correlations (with respect to fluctuations of classical random fields).

### Einstein's dreams

Albert Einstein did not believe in irreducible randomness, completeness of QM. He dreamed of a better, so to say "prequantum", model (Einstein and Infeld, 1961):

- 1). **Dream 1.** A mathematical model reducing quantum randomness to classical.
- 2). **Dream 2.** Renaissance of causal description.
- 3). **Dream 3.** Instead of particles, classical fields will provide the complete description of reality -- reality of fields (Einstein and Infeld, 1961):

" But the division into matter and field is, after the recognition of the equivalence of mass and energy, something artificial and not clearly defined. Could we not reject the concept of matter and build a pure field physics? What impresses our senses as matter is really a great concentration of energy into a comparatively small space. We could regard matter as the regions in space where the field is extremely strong. In this way a new philosophical background could be created."

The real trouble of the prequantum wave model (in the spirit of early Schrödinger) are not various NO-GO theorems, e.g., the Bell inequality (Khrennikov, 1999; 2008c; 2009a), but the problem which was recognized already by

Schrödinger. In fact, he gave up with his wave quantum mechanics, because of this problem: *A composite quantum system cannot be described by waves on physical space!* Two electrons are described by the wave function on  $R^6$  and not by two wave on  $R^3$ .

Einstein also recognized this problem (Einstein and Infeld, 1961):

"For one elementary particle, electron or photon, we have probability waves in a three - dimensional continuum, characterizing the statistical behavior of the system if the experiments are often repeated. But what about the case of not one but two interacting particles, for instance, two electrons, electron and photon, or electron and nucleus? We cannot treat them separately and describe each of them through a probability wave in three dimensions..."

### PCSFT

Einstein's Dreams 1 and 3 came true in PCSFT (but not Dream 2!) - a version of CSM in which fields play the role of particles. In particular, composite systems can be described by vector random fields, i.e., by the Cartesian product of state spaces of subsystems and not the tensor product. The basic postulate of PCSFT can be formulated in the following way:

A quantum particle is the symbolic representation of a "prequantum" classical field fluctuating on the time scale which is essentially finer than the time scale of measurements.

The prequantum state space  $M = L_2(R^3)$ , states are fields  $\phi: R^3 \rightarrow R$ ; "electronic field", "neutronic field", "photonic field" - classical electromagnetic field. An ensemble of "quantum particles" is represented by an ensemble of classical fields, probability measure  $\mu$  on  $M = L_2(R^3)$ , or random field  $\phi(x, \omega)$  taking values in  $M = L_2(R^3)$ . For each fixed value of the random parameter  $\omega = \omega_0$ ,  $x \rightarrow \phi(x, \omega_0)$  is a classical field on physical space.

### Density operator = covariance operator

Each measure (or random field) has the covariance operator, say  $D$ . It describes

correlations between various degrees of freedom.

The map  $\rho \mapsto D = \rho$  is one-to-one between density operators and the covariance operators of the corresponding prequantum random fields -- in the case of noncomposite quantum systems. In the case of composite systems this correspondence is really tricky.

Thus each quantum state (an element of the QM formalism) is represented by the classical random field in PCSFT. The covariance operator of this field is determined by the density operator. We also postulate that the prequantum random field has *zero mean value*. These two conditions determine uniquely Gaussian random fields. We restrict our model to such fields. Thus by PCSFT quantum systems are Gaussian random fields.

### Quantum observable = quadratic form

The map  $\hat{A} \rightarrow f_A(\phi) = (\hat{A}\phi, \phi)$  establishes one-to-one correspondence between quantum observables (self-adjoint operators) and classical physical variables (quadratic functionals of the prequantum field).

### Coincidence of averages

It is easy to prove that following equality holds:

$$E f_A(\hat{q}\omega) = \int_M f_A(\phi) d\mu(\phi) = \text{Tr} \rho \hat{A}.$$

In particular, for a pure quantum state  $\psi$ , consider the Gaussian measure with zero mean value and the covariance operator  $\rho = \psi \otimes \psi$  (the orthogonal projector on the vector  $\psi$ ), then

$$\int_M f_A(\phi) d\mu(\phi) = (\hat{A}\psi, \psi).$$

This mathematical formula coupling integral of a quadratic form and the corresponding trace is well known in measure theory. Our main contribution is coupling of this mathematical formula with quantum physics.

This is the end of the story for quantum noncomposite systems, *e.g.*, a single electron or photon (Khrennikov, 2005a; 2005b; 2006b-d).

### Beyond QM

In fact, PCSFT not only reproduces quantum averages, but it also provides a possibility to go beyond QM. Suppose that not all prequantum physical variables are given by QUADRATIC forms, consider more general model, all smooth functionals  $f(\phi)$  of classical fields. We only have the illusion of representation of all quantum observables by self-adjoint operators.

The map  $f \mapsto \hat{A} = f''(0) / 2$  projects smooth functionals of the prequantum field (physical variables in PCSFT) on self-adjoint operators (quantum observables). Then quantum and classical (prequantum) averages do not coincide precisely, but only approximately:

$$\int_M f_A(\phi) d\mu(\phi) = \text{Tr} \rho \hat{A} + O(t / T),$$

where  $T$  is the time scale of measurements and  $t$  the time scale of fluctuations of prequantum field. The main problem is that PCSFT does not provide a quantitative estimate of the time scale of fluctuations of the prequantum field. If this scale is too fine, *e.g.*, the Planck scale, then QM is “*too good approximation of PCSFT*”, *i.e.*, it would be really impossible to distinguish them experimentally. However, even a possibility to represent QM as the classical wave mechanics can have important theoretical and practical applications. And in the present paper we shall use the mathematical formalism of PCSFT to model brain's functioning. Although even in this case the choice of the scale of fluctuations is a complicated problem, we know that it is not extremely fine; so the model can be experimentally verified (in contrast to Roger Penrose we are not looking for cognition at the Planck scale!).

### 4. Composite systems

In CSM a composite system  $S = (S_1, S_2)$  is mathematically described by the Cartesian product of state spaces of its parts  $S_1$  and  $S_2$ . In QM it is described by the tensor product. Majority of researchers working in quantum foundations and, especially quantum information theory, consider this difference in the mathematical representation as crucial. In particular, entanglement which is a consequence of the

tensor space representation is considered as totally nonclassical phenomenon. However, we recall that Einstein considered the EPR-states as exhibitions of classical correlations due to the common preparation. PCSFT will realize Einstein's dream on entanglement.

Let  $S = (S_1, S_2)$ , where  $S_i$  has the state space  $H_i$  - complex Hilbert space. Then by CSM the state space of  $S$  is  $H_1 \times H_2$ . By extending PCSFT to composite systems we should describe ensembles of composite systems by probability distributions on this Cartesian product, or by a random field  $\phi(x, \omega) = (\phi_1(x, \omega), \phi_2(x, \omega)) \in H_1 \times H_2$ .

In our approach each quantum system is described by its own random field:  $S_i$  by  $\phi_i(x, \omega), i=1,2$ . However, these fields are CORRELATED - in completely classical sense. Correlation at the initial instant of time  $t=t_0$  propagates in time in the complete accordance with laws of QM. There is no action at the distance. It is a purely classical dynamics of two stochastic processes which were correlated at the beginning. (In fact, the situation is more complex: there is also the common random background, vacuum fluctuations; we shall come back to this question a little bit later).

### Operator realization of wave function

Consider now the QM-model, take a pure state case:  $\Psi \in H_1 \otimes H_2$ . Can one peacefully connect the QM and PCSFT formalisms? Yes! But  $\Psi$  should be interpreted in completely different way than in the conventional QM.

The main mathematical point:  $\Psi$  is not vector! It is an operator! It is, in fact, the non-diagonal block of the covariance operator of the corresponding prequantum random field:  $\phi(x, \omega) \in H_1 \times H_2$ . The wave function  $\Psi(x, y)$  of a composite system determines the integral operator:

$$\widehat{\Psi}\phi(x) = \int \Psi(x, y)\phi(y)dy.$$

We keep now to the finite-dimensional case. Any vector  $\Psi \in H_1 \otimes H_2$  can be represented in the form

$\Psi = \sum_{j=1}^m \psi_j \otimes \chi_j, \psi_j \in H_1, \chi_j \in H_2$ , and it determines a linear operator from  $H_2$  to  $H_1$

$$\widehat{\Psi}\phi = \sum_{j=1}^m (\phi, \chi_j)\psi_j, \phi \in H_2. \quad (2)$$

Its adjoint operator  $\Psi^*$  acts from  $H_1$  to  $H_2$ :

$$\widehat{\Psi}^*\psi = \sum_{j=1}^m (\psi, \psi_j)\chi_j, \psi \in H_1. \quad \text{Of course,}$$

$\widehat{\Psi}\widehat{\Psi}^* : H_1 \rightarrow H_1$  and  $\widehat{\Psi}^*\widehat{\Psi} : H_2 \rightarrow H_2$  and these operators are self-adjoint and positively defined. Consider the density operator corresponding to a pure quantum state,  $\rho = \Psi \otimes \Psi$ . Then the operators of the partial traces  $\rho^{(1)} \equiv \text{Tr}_{H_2}\rho = \widehat{\Psi}\widehat{\Psi}^*$  and  $\rho^{(2)} \equiv \text{Tr}_{H_1}\rho = \widehat{\Psi}^*\widehat{\Psi}$ .

### Basic equality

Let  $\Psi \in H_1 \otimes H_2$  be normalized by 1. Then, for any pair of linear bounded operators  $\widehat{A}_j : H_j \rightarrow H_j, j=1,2$ , we have:

$$\text{Tr}\widehat{\Psi}\widehat{A}_2\widehat{\Psi}^*\widehat{A}_1 = (\widehat{A}_1 \otimes \widehat{A}_2)_\Psi \equiv (\widehat{A}_1 \otimes \widehat{A}_2\Psi, \Psi). \quad (3)$$

This is a mathematical theorem (Khrennikov, 2009c); it will play a fundamental role in further considerations.

### Coupling of classical and quantum correlations

In PCSFT a composite system  $S = (S_1, S_2)$  is mathematically represented by the random field  $\phi(\omega) = (\phi_1(\omega), \phi_2(\omega)) \in H_1 \times H_2$ . Its covariance operator  $D$  has the block structure

$$D = \begin{pmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{pmatrix},$$

where  $D_{ii} : H_i \rightarrow H_i, D_{ij} : H_j \rightarrow H_i$ . The covariance operator is self-adjoint. Hence  $D_{ii}^* = D_{ii}$ , and  $D_{12}^* = D_{21}$ .

Here by the definition:  $(D_{ij}u_j, v_i) = E(u_j, \phi_j(\omega))(v_i, \phi_i(\omega)), u_i \in H_i, v_j \in H_j$ .

For any Gaussian random vector  $\phi(\omega) = (\phi_1(\omega), \phi_2(\omega))$  having zero average and any pair of self-adjoint bounded operators the following equality takes place:

$$\begin{aligned} \langle f_{A_1}, f_{A_2} \rangle &\equiv E f_{A_1}(\phi_1(\omega)) f_{A_2}(\phi_2(\omega)) \\ &= (\text{Tr} D_{11} \widehat{A}_1)(\text{Tr} D_{22} \widehat{A}_2) + \text{Tr} D_{12} \widehat{A}_2 D_{21} \widehat{A}_1. \end{aligned}$$

We remark that  $\text{Tr} D_{ii} \widehat{A}_i = E f_{A_i}(\phi_i(\omega)), i=1,2$ . Thus we have

$$f_{A_1} f_{A_2} = E f_{A_1} E f_{A_2} + \text{Tr} D_{12} \widehat{A}_2 D_{21} \widehat{A}_1. \quad \text{Consider a}$$

Gaussian vector random field such that  $D_{12} = \hat{\Psi}$ :

$$\begin{aligned} E(f_{A_1} - Ef_{A_1})(f_{A_2} - Ef_{A_2}) \\ = \langle \hat{A}_1 \otimes \hat{A}_2 \Psi, \Psi \rangle \equiv \langle \hat{A}_1 \otimes \hat{A}_2 \rangle_{\Psi}, \end{aligned} \quad (4)$$

or, for covariance of two classical random vectors  $f_{A_1}, f_{A_2}$ , we have:

$$\text{cov}(f_{A_1}, f_{A_2}) = \langle \hat{A}_1 \otimes \hat{A}_2 \rangle_{\Psi}.$$

We have the following equality for averages of quadratic forms of coordinates of the prequantum random field describing the state of a composite system:  $Ef_{A_i}(\phi)(\omega) = \text{Tr} D_{ii} \hat{A}_i$ . We want to construct a random field such that these averages will match those given by QM. For the latter, we have:

$$\begin{aligned} \langle \hat{A}_1 \rangle_{\Psi} &= \langle \hat{A}_1 \otimes I_2 \Psi, \Psi \rangle = \text{Tr}(\Psi \Psi^*) \hat{A}_1; \\ \langle \hat{A}_2 \rangle_{\Psi} &= \langle I_1 \otimes \hat{A}_2 \Psi, \Psi \rangle = \text{Tr}(\hat{\Psi}^* \hat{\Psi}) \hat{A}_2, \end{aligned}$$

where  $I_i$  denotes the unit operator in  $H_i, i=1,2$ . Thus it would be natural to take

$$D_{\Psi} = \begin{pmatrix} \hat{\Psi} \hat{\Psi}^* & \hat{\Psi} \\ \hat{\Psi}^* & \hat{\Psi}^* \Psi \end{pmatrix}.$$

However, *this operator is not positively defined!* It could not determine any probability distribution on the space of classical fields. We modify it to obtain a positively defined operator. Originally this modification had purely mathematical reasons, but there are deep physical grounds for it.

The operator

$$\tilde{D}_{\Psi} = \begin{pmatrix} \hat{\Psi} \hat{\Psi}^* + \varepsilon I & \hat{\Psi} \\ \hat{\Psi}^* & \hat{\Psi}^* \Psi + \varepsilon I \end{pmatrix}$$

is positively defined if  $\varepsilon > 0$  is large enough. Hence, it determines uniquely the Gaussian measure on the space of classical fields. Suppose now that  $\phi(\omega)$  is a random vector with the covariance operator  $\tilde{D}_{\Psi}$ . Then

$$\langle \hat{A}_1 \rangle_{\Psi} = Ef_{A_1}(\phi(\omega)) - \varepsilon \text{Tr} \hat{A}_1. \quad (5)$$

This relation for averages and relation (4) provide coupling between PCSFT and QM. Quantum statistical quantities can be obtained from corresponding quantities for

classical random field: *“irreducible quantum randomness” is reduced to randomness of classical prequantum fields.*

### Vacuum fluctuations

The additional term given by the unit operator in the diagonal blocks of the covariance operator of the prequantum vector field corresponds to the field of the white noise type. Such a field can be considered as vacuum fluctuations, vacuum field. PCSFT induces the following picture of reality:

Fluctuations of the vacuum field are combined with random fields representing quantum systems. Since we cannot separate, *e.g.*, electron from the vacuum field, we cannot separate totally any two quantum systems. Thus all quantum systems are “entangled” via the vacuum field. WHITE NOISE is the basis of everything in Nature - *Hida's Dream.*

### QM as renormalization formalism

Averages given by the mathematical formalism of traditional QM are obtained as renormalizations of classical averages, see (5). Thus the QM-formalism can be considered as a method of renormalization of averages with respect to vacuum fluctuations: it cancels the contribution of the vacuum field. Such a renormalization is especially important in the case of observables of the nontrace class. Here the contribution of the background field is infinite. Thus it should be subtracted from the classical average, cf. with renormalization procedures of QFT.

### Superstrong quantum correlations

In PCSFT such correlations (violating Bell's inequality) are due to the presence of the vacuum field. The off-diagonal term  $\hat{\Psi}$  can be so large only if the diagonal terms are completed by the contribution of the vacuum field. Mathematics tells us this. Thus they are so strong, because the vacuum field really couple any two systems; they are in the same fluctuating space.

Space is a huge random wave; quantum systems are spikes on this wave; they are correlated via this space-wave. Thus quantum correlations have two

contributions: 1) initial preparation; 2) coupling via the vacuum field.

The picture is pure classical. In this model the vacuum field is the source of additional correlations. It seems that this classical vacuum field is an additional (purely classical) quantum computational resource.

### 5. QL processing in the brain

Consider two time scales  $(t_c, t_{pc}) : t_{pc} \ll t_c$ , cognitive and precognitive. Take a signal in the brain oscillating on the time scale  $t_{pc}$ . We speculate that the brain has an integration device integrating these fluctuations over the interval  $t_c$ . The result of such integration is considered as a cognitive image. In this paper we do not present a concrete procedure of integration and, hence, creation of cognitive images from random fluctuations.

### 5.1 Multiplicity of time scales in brain and cognitive QLR

The main lesson from the experimental and theoretical investigations on the temporal structure of processes in brain is that there are various time scales. They correspond to (or least they are coupled with) various aspects of cognition. Therefore we are not able to determine once and for ever the cognitive time scale  $t_c$  ("psychological time"). There are a few such scales. We shall discuss some evident possibilities.

It is well known that there are well established time scales corresponding to the alpha, beta, gamma, delta, and theta waves. Let us consider these time scales as different cognitive scales.

For the alpha waves we choose its upper limit frequency, 12 Hz, and hence the  $t_{c,\alpha} \approx 0.083$  sec. For the beta waves we consider (by taking upper bounds of frequency ranges) three different time scales: 15 Hz,  $t_{c,\beta,low} \approx 0.067$  sec. - low beta waves, 18Hz,  $t_{c,\beta} \approx 0.056$  sec. - beta waves, 23 Hz  $t_{c,\beta,high} \approx 0.043$  sec. - high beta waves. For gamma waves we take the characteristic frequency 40 Hz and hence the time scale  $t_{c,\gamma} \approx 0.025$  sec.

### 5.2 Precognitive time scale

Our choice of the precognitive (very fine) time scale  $t_{pc}$  will be motivated by so called *Taxonomic Quantum Model*, see proposed by Geissler and collaborators (Schack *et al.*, 2001), for representation of cognitive processes in the brain (which was developed on the basis of the huge experimental research on time-mind relation. They found that information processing in cognitive tasks is based on time scales  $Q_q = q \times Q_0$ , where  $s_{pc} = Q_0 = 4.6ms$ . We choose  $Q_0$  as the unit of the precognitive time scale. This corresponds to frequencies  $\approx 220$  Hz. Under such an assumption about the precognitive scale we can find the measure of QL-ness for different EEG bands. For the alpha scale, we have

$$\kappa_\alpha = \frac{Q_0}{t_{c,\alpha}} \approx 0.055.$$

For the beta scales, we have:

$$\begin{aligned} \kappa_{c,\beta,low} &= \frac{Q_0}{t_{c,\beta,low}} \approx 0.069; \kappa_{c,\beta} \\ &= \frac{Q_0}{t_{c,\beta}} \approx 0.082; \kappa_{c,\beta,high} = \frac{Q_0}{t_{c,\beta,high}} \approx 0.107. \end{aligned}$$

For the gamma scale we have:

$$\kappa_\gamma = \frac{Q_0}{t_{c,\gamma}} \approx 1.84.$$

Smaller  $\kappa$  correspond to larger integration time in the process of creation of cognitive images; less images can be created and processed. "Thinking through the alpha waves" is essentially less advanced than, e.g., "thinking through the gamma waves".

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