A Novel Nature Inspired Fuzzy Tsallis Entropy Segmentation of Magnetic Resonance Images

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ABSTRACT

Medical imaging comprises a large number of non-invasive techniques to assist diagnosis or treatment of different medical conditions. Magnetic resonance imaging (MRI) is a noninvasive imaging technique which has a wide range of applications such as neuroimaging, cardiac magnetic resonance imaging, spinal imaging, liver and gastrointestinal imaging, functional magnetic resonance imaging etc. Neuroimaging helps the physician to investigate functions of the brain or neurological disorder. In this paper, a novel quantum optimization technique referred as Improved Quantum Particle Swarm Optimization (IQPSO) for optimizing three levels Fuzzy Tsallis Entropy (FTE) is proposed for segmenting brain MR images. Fuzzy Tsallis Entropy (FTE) is a fuzzy based threshold method segments the image based on entropies. The proposed method is validated on the data sets obtained from the Chennai scan center and is compared with other standard optimization methods such as Particle Swarm Optimization (PSO) and Quantum Particle Swarm Optimization (QPSO). The analysis shows that FTE optimized using Improved QPSO produces maximum entropy of 33.7623, 32.9868, 36.0231, 36.1231, 40.9789, 40.9789, 40.9789, 41.231 , 41.2314, 43.6994 for the brain MRI slices 13,12,11,10,9,8,7,6,5 and 1 of patient 1 and 13.006, 13.126, 13.126, 12.673, 12.295, 12.202, 11.229 for the brain MRI slices 1,2,3,4,5,6 and 7 of patient 2. The obtained results are better than traditional Particle Swarm Optimization, IQPSO and FTE.

Key Words: Fuzzy Tsallis Entropy, Improved Quantum Particle Swarm Optimization, Magnetic Resonance Imaging, Particle Swarm Optimization and Quantum Particle Swarm Optimization

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1. Introduction

Medical image processing is one of the techniques to visualize and analysis the diverse types of images from user perspective which in turn help the medical practitioner to diagnose or examine the disease. There are numerous image processing methods to analyze different types of medical images and to develop user interactive medical image processing applications. Medical image segmentation is a preprocessing technique of medical image analysis to identify or locate objects present in an image. A number of methods proposed for segmentation of images. They are classified as thresholding and clustering based methods, histogram based methods, edge detection methods and region growing methods.

Thresholding is a widely used segmentation method due to its simplicity and computational efficiency and classified as global threshold (Otsu, 1975), edge based (Canny, 1986), Sobel Lapalacian (Gonzales *et al.*, 2002), region based techniques (Pavlidis *et al.*, 1974) and hybrid techniques like Watershed algorithm (Roerdink *et al.*, 2001). These methods operate on pixel intensities. Image

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pixels whose intensities are greater than the given threshold are grouped into one class and the remaining pixels are moved into another class. There are various diagnostic imaging techniques such as magnetic resonance imaging (MRI), ultrasound, x-ray, positron emission tomography etc. MRI is a commonly used imaging modality to detect cancers, cysts, tumors and sometimes it detects breast cancer missed by mammography. Medical image segmentation studies objects present in the image and identifies them as normal and abnormal regions.

Swarm Intelligence (SI) is a nature inspired approach which emphasizes the collective or group behavior of animals. SI based multi-agent system consists of a group of agents which interact with each other and their environment. This system governs two principles; self-organization and stigmergy. SI is an emerging research area being applied in interdisciplinary approaches due to its simplicity, robustness, adaptability and scalability. The system consists of a number of optimization algorithms such as PSO, Ant Colony Optimization (ACO) spider colony and bacterial colony applied in the areas of data mining, neural network, bioinformatics, biomedical and pharmaceutical problems etc. (Nandita *et al.,* 2011; Wen-Bing *et al.,* 2011; 2007). Its medical applications include tumor detection, gene clustering, drug design, biometrics and cancer detection and classification (Chengzhong *et al.,* 2008; Anusuya *et al.,* 2008). These algorithms have drawbacks like tuning of parameters based on problem, time critical analysis and the stagnation of agents or particles.

In this paper, a novel method of thresholding called optimization of three levels FTE using Improved QPSO (IQPSO) is proposed. The levels of threshold are chosen based on membership values of features present in the image using FTE. The rest of the paper is organized as follows. Section 2 describes the work related to the proposed method. Three level Fuzzy Tsallis is elaborated in section 3. QPSO is dealt in section 4 while in section 5 optimization of FTE using IQPSO with its algorithm are discussed. Experimental results and comparative performances are provided in section 6. The conclusions of the research are presented in section 7.

2. Related Work

A good thresholding method maximizes the uniformity of pixels within the regions and minimizes the uniformity across regions. Fuzzy based approaches perform well in automatic selection of optimal threshold (Wen-Bing *et al.,* 2003; Souad *et al.,* 2005) and provides much information content of an object when an image transformed to fuzzy domain. The authors insist that three level thresholding produce better results than midpoint thresholding methods. The technique of Fuzzy Entropy was introduced by Luca and Termini in 1979.

Fuzzy Entropy is computed on 2D histogram to obtain various levels of threshold where genetic algorithm is employed to detect parameters for Fuzzy Entropy (Jinhui, 2013). The CT image of Human head is segmented using biogeography based optimization (Amitava, 2012) by the principle of 'fuzzy entropy maximization'. Particle Swarm Optimization has been introduced to find the fuzzy threshold and parameters for fuzzy entropy. The results showed that the search ability of the method is superior to exhaustive search method in segmenting remote sensing images (Linyi *et al.,* 2008). Performance analysis between GA-Fuzzy and PSO-Fuzzy (Amitava, 2012) showed that PSO–Fuzzy outperforms GA-Fuzzy technique even when Signal to Noise Ratio (SNR) is low (Du Feng *et al.,* 2005). QPSO was proposed to improve the search ability of PSO and is used to find global optimal threshold on images (TIAN, 2009).

Three level thresholding using Maximum Fuzzy Entropy has been demonstrated in the application of image segmentation (Wen-Bing *et al.,* 2003). A method of clustering was used to segment complicated objects present in breast cancer images (Kun Wang *et al.,* 2009). ACO is applied to find the optimal threshold for Fuzzy entropy method (Wen-Bing *et al.,* 2007). Tsallis entropy is an extension of generalized entropy proposed by Tsallis. The implementation of FTE is simple and it performed well in threshold selection of image segmentation (Portes *et al.,* 2004). FTE optimization using PSO is proposed (Yinggan, 2008) to improve the performance of Shannon Entropy. In this work, Fuzzy inspired Tsallis Entropy was introduced to handle uncertainty information content in medical images. It has been observed from the literature that the selection of threshold from the optimized entropy methods would produce better results than the traditional methods. In this paper, the applicability of standard PSO was tested on the real time images and a novel algorithm has been proposed to optimize the threshold value obtained using three level Fuzzy Tsallis Entropy.

3. Fuzzy Tsallis Entropy (FTE)

A grayscale image I of size MxN can be defined as I(x,y) ϵ k; $I(x,y) \in k, \forall (x,y) \in p$

Where $y = 0,1 \ldots N-1$, $x = 0,1 \ldots M-1$, and $k=\{0,1,\ldots L-1\}$, L being the maximum intensity of gray scale image. The histogram of the image is represented as $H=\{h_0,h_1,\ldots h_{L-1}\}$ where h_n denotes the number of pixels with gray level n in k_n.

$$\sum_{k=0}^{L-1} h_{mn} = 1 \qquad h_{mn} = \frac{h_n}{M \times N}$$

$$0 \le h_{mn} \le 1 \qquad n = 0,1,\ldots L-1$$

Where h_{mn} represents the histogram of an image. The original image of size MxN is divided into three regions (dark, gray and bright) such as R_d, R_g and R_b and its probability distribution are given as $p_d=P(R_d)$, $p_g=P(R_g)$ and $p_b=P(R_b)$. The membership functions of these regions are considered as $\mu_d(n)$, $\mu_g(n)$ and $\mu_b(n)$. The region dark has low intensities, the region bright has high intensities and the intensities of gray lies between a maximum of dark and minimum of bright. The membership function of these three regions is denoted as

$$\mu_d(n) = \begin{cases} 1, & n \le a1 \\ 1 - \dfrac{(n-a_1)^2}{(c_1-a_1)\times(b_1-a_1)}, & a1 < n \le b1 \\ \dfrac{(n-c_1)^2}{(c_1-a_1)\times(c_1-b_1)}, & b1<n\le c1 \\ 0, & n>c1 \end{cases} \qquad \textbf{(Eq.1)}$$

$$\mu_g(n) = \begin{cases} 0, & n \le a1, \\ \dfrac{(n-a_1)^2}{(c_1-a_1)\times(b_1-a_1)}, & a1< n \le b1 \\ 1 - \dfrac{(n-c_1)^2}{(c_1-a_1)\times(c_1-b_1)}, & b1<n\le c1 \\ 1 & c1<n\le a2 \\ 1 - \dfrac{(n-a2)^2}{(c_2-a_2)\times(b_2-a_2)}, & a2<n\le b2 \\ \dfrac{(n-c_2)^2}{(c_2-a_2)\times(c_2-b_2)} & b2<n\le c2 \\ 0 & n>c2 \end{cases} \qquad \textbf{(Eq.2)}$$

$$\mu_b(n) = \begin{cases} 0, & n \le a2, \\ \dfrac{(n-a_2)^2}{(c_2-a_2)\times(b_2-a_2)}, & a2 < n \le b2 \\ 1 - \dfrac{(n-c_2)^2}{(c_2-a_2)\times(c_2-b_2)} & b2<n\le c2 \\ 1 & n>c2 \end{cases} \qquad \textbf{(Eq.3)}$$

The parameters in equation (1), (2) and (3) satisfies the condition $0 \le a1 \le b1 \le c1 \le a2 \le b2 \le c2 \le 255$. Fuzzy Tsallis entropy for the regions R_d, R_g and R_b is written as

$$H_d = \frac{1}{\alpha - 1}\left[1 - \sum_{n=0}^{p-1}\left(\frac{p_n \times \mu_d(n)}{p_d}\right)^{\alpha}\right] \quad \textbf{(Eq.4)}$$

$$H_g = \frac{1}{\alpha - 1}\left[1 - \sum_{n=p}^{q-1}\left(\frac{p_n \times \mu_g(n)}{p_g}\right)^{\alpha}\right] \quad \textbf{(Eq.5)}$$

$$H_b = \frac{1}{\alpha - 1}\left[1 - \sum_{n=q}^{L-1}\left(\frac{p_n \times \mu_b(n)}{p_b}\right)^{\alpha}\right] \quad \textbf{(Eq.6)}$$

Hence, the total fuzzy Tsallis entropy is written as

$$H = H_d + H_g + H_b + (1-\alpha) * H_d * H_g * H_b \quad \textbf{(Eq.7)}$$

And the objective function is given as

$$H_{obj} = \max\{H\} \quad \textbf{(Eq.8)}$$

Where the parameter α is system dependent real number, p_n denotes the frequency of occurrence of gray level n and the thresholds p and q are found to segment the image into three regions. The membership functions of an image discussed in equation (1), (2) and (3) are shown in Figure 1.

4. Quantum Particle Swarm Optimization (QPSO)

Swarm intelligence algorithms, also referred to as nature-inspired algorithms are motivated by social behavior of animals and have proved to be very efficient in solving real world optimization problems. These algorithms include PSO, ACO, Stochastic Diffusion Search, and Bacterial Foraging and so on. Quantum Particle Swarm Optimization (QPSO) is a variant of PSO (Kennedy, 1995) and is proposed for better search ability and good convergence speed. PSO has the limitation of converging at local optima, to improve the searching ability of PSO, QPSO and Chaos theory based application is introduced on the image matching problem (Maolong, 2008). Quantum theory introduced into PSO and a new algorithm called QPSO was proposed (Jun *et al.,* 2004). QPSO guaranteed optimal solution, unlike PSO as no velocity vectors are needed and takes the least number of parameters to adjust. QPSO has been applied in a wide range of optimization problems (Leandro *et al.,* 2010; Debao *et al.,* 2012). A variant of QPSO called Multi Elicit QPSO (MEQPSO) on gene expression analysis is tested and superiority of MEQPSO is proved (Jun *et al.,* 2012b).

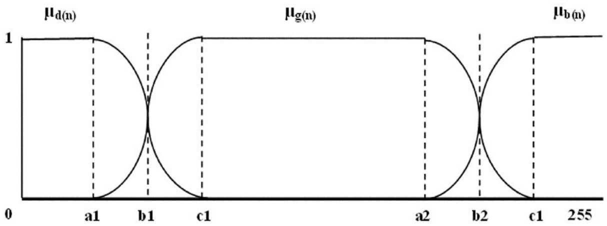


Figure 1. Membership functions of dark, gray and bright regions of an image.

Effectiveness of QPSO on time complexity and convergence rate are analyzed and improvements in these factors are proposed through algorithms such as QPSO with random mean position (Maolong *et al.,* 2008), QPSO with ranking operator and Multi-Elitist QPSO (Jun *et al.,* 2012a).

In the model of QPSO, the particles are moving in quantum space and the state of a particle is characterized by the wave function $\Psi(x,t): \left|\Psi_{(x,t)}\right|^2$ is the probability density function which denotes the probability of particles appearing in a certain position and particles changes its position according to the following iterative equations; equation (9) and equation (10).

$$x_i(t+1) = P_i + \beta\left|mbest_i - x_i(t)\right| * \ln(1/u),$$
$$\text{If } k \ge 0.5 \quad \textbf{(Eq.9)}$$

$$x_i(t+1) = P_i - \beta\left|mbest_i - x_i(t)\right| * \ln(1/u),$$
$$\text{If } k < 0.5 \quad \textbf{(Eq.10)}$$

where β is called Contraction–Expansion coefficient to control convergence speed of the algorithm. M is the population size, P_i is the personal best (*pbest*) positions of particles given in equation (11) and *mbest* is the mean of *pbest* values given in equation (12).

$$P_i = \varphi * pbest_i + (1-\varphi) * gbest_i \quad \textbf{(Eq.11)}$$

$$mbest = (1/M\sum_{i=1}^{M} Pi1, 1/M\sum_{i=1}^{M} Pi2, \ldots\ldots, 1/M\sum_{i=1}^{M} PiD) \quad \textbf{(Eq.12)}$$

The parameters k, u and φ are random numbers distributed in the range [0,1]. The procedure of QPSO is given below;

Initialize population size M, the positions of particles X and Dimension D.
Evaluate fitness of all particles.
Calculate pbest (P_{id}) positions of all particles and find gbest (P_{gd}) among particles.
While (condition)
Calculate mbest positions according to equation (12);
For i= 1 to Population Size (M)
For j = 1 to dimension D
update particles positions as per equation (9) and equation (10);
End for j;
End for i;
End While

5. Proposed Work

Improved QPSO is proposed by imposing refinement in mean best position (*mbest*) and personal best position (*pbest*) of traditional QPSO. In general, the mean best position influences searching ability of particles, so detailed attention is needed to be given when calculating the value of *mbest* (Maolong, 2008). This is done by ranking the particles according to its fitness. If a particle has higher fitness value then the particle is the elitist one among other particles. Thus, the mean best position is updated as

$$mbest = 1/M\sum_{i=1}^{M} a_{i1}P_{i1}, \frac{1}{M}\sum_{i=1}^{M} a_{i2}P_{i2}, \ldots\ldots, 1/M\sum_{i=1}^{M} a_{iD}P_{iD}) \quad \textbf{(Eq.13)}$$

where $a_{i1}, a_{i2} \ldots a_{iD}$ are weight coefficients linearly decreases with rank, i.e., highest value is multiplied with a particle whose fitness is high. In this paper weight factor decreases linearly from 1.5 to 0.5.

Sometimes, at the later stage of the search process, the *gbest* position obtained may belong to local optimum point which leads to premature convergence. In order to avoid such cases, the *pbest* position of a particle is updated by its own *pbest* and *pbest* of a randomly selected particle (Jun, 2012a). The random selection of particle is associated with ranking based on particle's fitness. The selection probability of random particle q is written as

$$PS(q) = \begin{cases} \dfrac{2 * rank_q}{rank_i(rank_i - 1)}, rank_q > rank_i \\ 0, rank_q \prec rank_i \end{cases} \quad \textbf{(Eq.14)}$$

where $rank_i$ is the rank of particle i and $rank_q$ is the rank of randomly selected particle with a constraint $rank_q > rank_i$, the sum of the selection probabilities of all candidates equals 1.

Procedure of proposed FTEIQPSO

Initialize population size M, the positions of particles X and Dimension D
Evaluate fitness of all particles as per equation (8)
Calculate *pbest* (P_{id}) positions of all particles and find *gbest* (P_{gd}) among particles.
While (condition)
Calculate *mbest* positions according to equation (13)
For i= 1 to Population Size (M)
Rank the particles with their fitness, select a particle q ($rank_q > rank_i$) and update the *pbest*(p_{id}) position of particle based on its own *pbest*$_i$ and *pbest*$_q$
For j = 1 to dimension D
Update positions as per the following equations

$$p[i][j] = \eta * P[i][j] + (1-\eta) * P_q[i][j]; \quad \eta = rand[0,1]$$

If rand()<0.5

$$x_{i,j}(t+1) = P[i][j] + \beta\left|mbest - x_{i,j}(t)\right| * \ln(1/u)$$

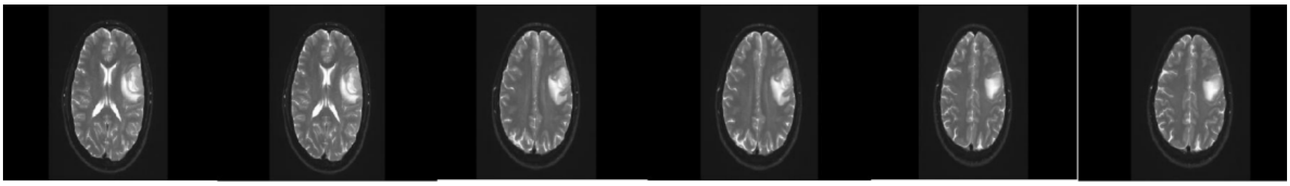
Else

$$x_{i,j}(t+1) = P[i][j] - \beta\left|mbest - x_{i,j}(t)\right| * \ln(1/u)$$

End if
End for
Evaluate fitness and update *pbest* and *mbest* positions
End For
End While
End.

6. Experimental Results

In this paper, a novel method of thresholding to segment brain MR images, and to locate the abnormal regions present in those images is proposed. The proposed algorithm was tested on the clinical datasets acquired from a diagnostic center. The data sets are T2 weighted FS axial brain images of size 650x1105 with various slices of two patients. Each image has a defected region of various sizes and those regions are referred as Region of Interests (ROI). The ROIs are exactly separated by optimal thresholds values obtained by Fuzzy Tsallis Entropy.

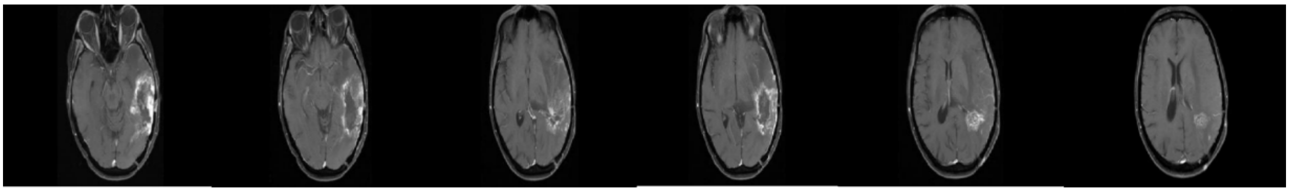


2.a 2.b 2.c 2.d 2.e 2.f

Figures 2. a-f MRI Slices (8-13) of brain (Patient 1).

3.a 3.b 3.c 3.d 3.e 3.f

Figures 3. a-f. Optimal results of FTEIQPSO on the images 2.a to 2.f.

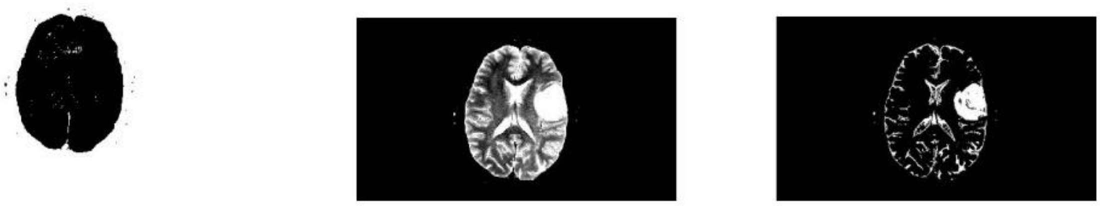


4.a 4.b 4.c 4.d 4.e 4.f

Figures 4. a-f. MRI Slices (2-7) of brain (Patient 2).

5.a 5.b 5.c 5.d 5.e 5.f

Figures 5. a-f. Optimal results of FTEIQPSO on the images 4.a to 4.f.



6.a 6.b 6.c

Figure 6. Different levels of image segmentation (Slice 8 of brain image of Patient1) 6.a Th=(p=0, q=82), 6.b Th=(p=83, q=136), 6.c Th=(p=137, q=255).

7.a 7.b 7.c

Figure 7. Different levels of image segmentation (Slice 2 of brain image of Patient2). 7.a Th(p=0, q=72), 7.b Th=(p=73, q=125), 7.c Th(p=126, q=255).

Table 1. Comparative Results.

Test Image (Patient 1)	Threshold				Comp. Time				Fuzzy Max Entropy		
	FTE	FTE-PSO	FTE-QPSO	FTE IQPSO	FTE	FTE-PSO	FTE-QPSO	FTE-IQPSO	FTE	FTE-PSO	FTE-QPSO
Slice 13	141	143	189	189	1.2069	3.8138	3.8055	3.405	10.14	19.677	32.5868
Slice12	141	143	142	143	1.2198	3.7994	3.7449	3.145	10.14	19.677	32.5868
Slice11	136	140	136	138	0.7533	3.8085	3.6423	3.242	10.19	22.224	36.0231
slice10	136	140	136	139	0.8175	3.8060	3.2637	3.019	10.19	22.224	36.0231
Slice9	133	140	131	135	0.8533	3.6762	3.5673	3.457	10.05	22.224	37.84534
Slice8	133	140	131	136	0.8310	3.8762	2.9872	2.085	10.05	22.224	37.84534
Slice7	133	134	131	135	0.7728	4.1350	3.5472	3.234	10.05	25.56	38.84534
Slice6	133	134	132	137	0.7975	4.4422	3.5472	3.438	10.05	25.56	39.6751
Slice5	133	134	134	139	0.8020	3.8795	3.5472	3.438	10.05	25.67	39.6751
Slice1	142	143	144	147	0.8070	3.8111	3.6072	3.438	9.924	27.019	42.8734
Test Image (Patient 2)											
Slice1	127	137	126	130	0.7078	3.8508	3.7534	2.988	10.31	12.921	13.0056
slice2	129	130	123	129	0.8124	3.7872	3.7534	2.988	10.25	12.848	12.8933
Slice3	139	139	136	139	0.6989	3.7906	3.1534	3.154	10.04	12.559	12.8933
Slice4	128	136	125	129	0.7482	3.8053	3.7265	2.79	10.05	12.559	11.253
Slice5	152	152	152	149	0.6860	3.8203	3.1871	2.614	9.794	12.176	12.2951
Slice6	131	131	131	135	0.7603	3.8331	3.4712	2.516	9.697	11.968	9.87261
Slice7	153	153	151	151	0.6132	3.7928	3.1091	2.017	8.823	10.061	11.0012

Table 4. Parameters set for various methods.

Method	Parameters
FTEPSO	c1=2;c2=2;wmax=0.9;wmin=0.4;popsize=20,α=0.5
FTEQPSO	popsize=20;MAXITER=100;dimension=2;α=0.5
FTEIQPSO	popsize=20;MAXITER=100; dimension=2; decreasing weight factor=1.5 to 0.5;α=0.5

Table 2. Results of FTEIQPSO for Patient 1.

Patient-1		
Test Images	Fuzzy Max Entropy FTEIQPSO	mbest
Slice 13	33.7623	150
Slice12	32.9868	139
Slice11	36.0231	136
Slice10	36.1231	137
Slice9	40.9789	165
Slice8	40.9789	165
Slice7	40.9789	150
Slice6	41.2314	165
Slice5	41.2314	172
Slice1	43.6994	195

7. Conclusion

Detection of the desired object from an image using thresholding is a big challenge. Fuzzy entropy techniques are found to be performed well to deal with uncertainty information of images and identifies much information than other entropy methods. In this work, an improved QPSO to optimize entropy values of Fuzzy Tsallis Entropy to segment and isolate specific regions of MR images has been proposed. Analysis of the experimental results shows that the performance of the proposed method is better than other existing methods in literature and the results produced are 30%, 15% and 3% better than FTE, FTEPSO and FTEQPSO respectively.

Table 3. Results of FTEIQPSO for Patient 2.

Patient-2		
Test Images	Fuzzy Max Entropy FTEIQPSO	mbest
Slice1	13.006	133
slice2	13.126	121
Slice3	13.126	140
Slice4	12.673	110
Slice5	12.295	105
Slice6	12.202	133
Slice7	11.229	140

References

Amitava C, Patrick S, Amir N, Raphael B. An improved biogeography based optimization approach for segmentation of human head CT-scan images employing fuzzy entropy. Engineering Applications of Artificial Intelligence 2012; 25(8): 1698-1709.

Anusuya V, Latha P. Medical image thresholding using WQPSO and maximum entropy. Proc of Int Conf on Advances in Computing, Communications and Informatics 2012, India. 1219-1224.

Canny J. A Computational Approach to Edge Detection. Proc IEEE Trans on Pattern Analysis and Machine Intelligence 1986; 8(6): 679–698.

Chengzhong H, Bin Y, Hua J, Dahui W. MR Image Segmentation Based On Fuzzy C-Means Clustering and the Level Set Method. Proc of IEEE Int Conf on Fuzzy Systems and Knowledge Discovery 2008; Shandong 1:67–71.

Debao C, Jiangtao W, Feng Z, Weibo H, Chunxia Z. An improved group search optimizer with operation of quantum-behaved swarm and its application 2012; Applied Soft Computing 12(2): 712–725.

Du F, Shi W, Chen L, Deng Y, Zhu Z. Infrared image segmentation with 2-D maximum entropy method based on particle swarm optimization (PSO). Pattern Recognition Letters 2005; 26(5):597-603.

Woods G. Digital Image Processing 3rd Ed. (DIP/3e), 2009.

Jinhui L, Yiliang Z. Multi threshold image segmentation using maximum fuzzy entropy based on a new 2D histogram. Optik 2013; 124(18): 3756-3760.

Jun S, Wei C, Wei F, Xiaojun W, Wenbo X. Gene expression data analysis with the clustering method based on an improved quantum-behaved Particle Swarm Optimization. Engineering Applications of Artificial Intelligence 2012; 25(2):376–391.

Jun S, Xiaojun W, Vasile P, Wei F, Choi-Hong L, Wenbo X. Convergence analysis and improvements of quantum-behaved particle swarm optimization. Information Sciences 2012; 193(15): 81-103.

Jun S, Bin F, Wenbo X. Particle swarm optimization with particles having quantum behaviour. Congress on Evolutionary Computation, China. 2004; 1:325 – 331.

Kennedy J, Eberhart R. Particle swarm optimization. Proc IEEE Int Conf on Neural Networks 1995; 4:1942–1948.

Kun W, Zhihui D, Yinong C, Sanli L. V3COCA: An effective clustering algorithm for complicated objects and its application in breast cancer research and diagnosis. Simulation Modelling Practice and Theory 2009; 17(2): 454-470.

Leandro D, Santos C. Gaussian quantum-behaved particle swarm optimization approaches for constrained engineering design problems. Expert Systems with Applications 2010;37(2): 1676–1683.

Linyi L, Deren L. Fuzzy entropy image segmentation based on particle swarm optimization. Progress in Natural Science 2008; 18(9): 1167-1171.

Maolong X, Jun S, Wenbo X. An improved quantum-behaved particle swarm optimization algorithm with weighted mean best position. Applied Mathematics and Computation 2008; 205 (2): 751–759.

Nandita S, Amitava C, Sugata M. An adaptive bacterial foraging algorithm for fuzzy entropy based image segmentation. Expert Systems with Applications 2011; 38(12): 15489–15498.

Otsu N. A Threshold Selection Method from Gray-Level Histograms. IEEE Trans on Systems, Man, and Cybernetics 1979; 9(1): 62-66.

Pavlidis T, Horowitz SL. Segmentation of Plane Curves. IEEE Trans on Computers 1974; 23(8):860-870.

Portes M, Esquef IA, Gesualdi AR. Image thresholding using Tsallis entropy. Pattern Recognition Letters 2004; 25 (9): 1059–1065.

Roerdink J, Meijster A. The Watershed Transform: Definitions, Algorithms and Parallelization Strategies. Fundamenta Informaticae 2001; 41: 187-228.

Souad B, Mohammed B. Recursive algorithm based on fuzzy 2-partition entropy for 2-level image thresholding. Pattern Recognition 2005; 38(8): 1289–1294.

Tian J, Zeng J. 2D Fuzzy Maximum Entropy Image Threshold Segmentation Method Based on QPSO. Computer Engineering 2009; 35(3): 230-232.

Wen-Bing T, Hai J, Liman L. Object segmentation using ant colony optimization algorithm and fuzzy entropy. Pattern Recognition Letters 2007; 28(7): 788-796.

Wen-Bing T, Jin-Wen T, Jian L. Image segmentation by three-level thresholding based on maximum fuzzy entropy and genetic algorithm. Pattern Recognition Letters 2003; 24(16): 3069–3078.

Yinggan T, Qiuyan D, Xinping G, Fucai L. Threshold Selection Based on Fuzzy Tsallis Entropy and Particle Swarm Optimization. NeuroQuantology 2008; 6(4): 412-419.