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A Brief Note on Time Evolution of Quantum Wave Function and of Quantum Probabilities During Perception and Cognition of Human Subjects

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Abstract
We apply our theory of quantum time dynamics of cognitive entities to experimental data in order to calculate the time evolution of quantum wave function and of quantum probabilities during cognitive processes of the human subject. The results are used in order to elucidate the mind mechanism of human decision under the perspective of quantum mechanics.

Key Words: quantum cognitive process, wave function, quantum probability, time evolution of quantum systems, quantum dynamics of cognitive entities, human perception, cognitive process

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1. Introduction

Application of Quantum Mechanics to Cognitive Processes in Psychology

The aim of the present paper is to estimate for the first time the time evolution of the quantum wave function and of quantum probabilities during the process of perception-cognition of a human subject, and to give on this basis an explanation in quantum mechanical terms of such basic mind mechanism. The results are obtained on the basis of a previous performed experiment. The theory on this matter was formulated by us in 2005 (Conte, 2007). In a number of previous papers (Conte, 2002, 2003, 2006, 2008), we also exposed the features of our formulation that is based on the statement that quantum mechanics is a "physical" theory of cognitive processes of the mind.

2. The Theoretical Background

The experiment of perception-cognition was performed on a group of 72 subjects as it was described in our paper (Conte, 2009). To summarize here, a quantum dichotomous mental observable, $B = \pm$, was measured and it was considered a quantum wave function of a superposition of states

$$\psi = c_+\varphi_+ + c_-\varphi_- \quad \textbf{(2.1)}$$

with

$$|c_+|^2 + |c_-|^2 = 1,\ \varphi_+ = \begin{pmatrix}1\\0\end{pmatrix},\ \varphi_- = \begin{pmatrix}0\\1\end{pmatrix} \quad \textbf{(2.2)}$$

The experimental probabilities were obtained

$P_+ = P(B=+) = |c_+|^2$ =0.6667

and

$$P_- = P(B=-) = |c_-|^2 = 0.3333 \quad \textbf{(2.3)}$$

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The problem to study the time evolution of such a quantum system in psychology during cognition of a human subject, was previously considered by D. Aerts (Arts, 2003), by A. Khrennikov (2003), by J. R. Busemeyer (2006 and following papers). Based on a previous approach of C. Altafini (Altafini, 2003; Magnus, 1954), we developed in 2005 our theory of quantum time dynamics of cognitive entities. In appendix A we report an exposition of such theory. In this paper we are interested to the application of some basic formulas that are given in the following manner.

The Hamiltonian H of the cognitive entity, as derived by this theory, is fully linear time invariant (see the equation 3.3) and its exponential solution will take the following form

$$e^{t(\sum_{j=1}^{3}(a_j+b_j)A_j)} = \cos(\frac{k}{2}t)I + \frac{2}{k}\text{sen}(\frac{k}{2}t)\left(\sum_{j=1}^{3}(a_j+b_j)A_j\right) \quad \textbf{(2.4)}$$

with

$$k=\sqrt{(a_1+b_1)^2+(a_2+b_2)^2+(a_3+b_3)^2}\ . \quad \textbf{(2.5)}$$

Still, it will result that

$$U(t)=\begin{pmatrix} \cos\frac{k}{2}t+\frac{i}{k}\text{sen}\frac{k}{2}t(a_3+b_3) & \frac{1}{k}\text{sen}\frac{k}{2}t[a_2+b_2+i(a_1+b_1)] \\ \frac{1}{k}\text{sen}\frac{k}{2}t[-a_2-b_2+i(a_1+b_1)] & \cos\frac{k}{2}t-\frac{i}{k}\text{sen}\frac{k}{2}t(a_3+b_3) \end{pmatrix} \quad \textbf{(2.6)}$$

and, obviously, it will result to be uni-modular as required. This is the matrix representation of time evolution operator for the considered cognitive entity.

The expression of the state $\psi(t)$, the quantum wave function of the cognitive entity at any time, will be given in the following manner

$$\psi(t)= \begin{bmatrix} c_+\left[\cos\frac{k}{2}t+\frac{i}{k}sen\frac{k}{2}t(a_3+b_{3)}\right] \\ +c_-\left[\frac{1}{k}sen\frac{k}{2}t\left[(a_2+b_2)+i(a_1+b_1)\right]\right] \end{bmatrix}\phi_+ + \begin{bmatrix} c_+\left[\frac{1}{k}sen\frac{k}{2}t\left[i(a_1+b_1)-(a_2+b_2)\right]\right] \\ +c_-\left[\cos\frac{k}{2}t-\frac{i}{k}sen\frac{k}{2}t(a_3+b_3)\right] \end{bmatrix}\phi_- \quad \textbf{(2.7)}$$

Finally, the two probabilities $P_+(t)$ and $P_-(t)$ that are expected for future selection and decision to $B=\pm$, as consequence of cognition measurement and context influence, will be given at any time t by the following expressions

$$P_+(t)=(A^2+B^2)\cos^2\frac{k}{2}t + \frac{1}{k^2}sen^2\frac{k}{2}t(P^2+Q^2)+\frac{senkt}{k}(AP+BQ) \quad \textbf{(2.8)}$$

and

$$P_-(t)=(C^2+D^2)\cos^2\frac{k}{2}t + \frac{1}{k^2}sen^2\frac{k}{2}t(S^2+R^2)+\frac{senkt}{k}(RC+DS)$$

where

$A=\text{Re } c_+$, $B=\text{Im } c_+$, $C=\text{Re } c_-$, $D=\text{Im } c_-$,
$P=-D(a_1+b_1)+C(a_2+b_2)-B(a_3+b_3)$,
$Q=C(a_1+b_1)+D(a_2+b_2)+A(a_3+b_3)$, **(2.9)**
$R=-B(a_1+b_1)-A(a_2+b_2)+D(a_3+b_3)$,
$S=A(a_1+b_1)-B(a_2+b_2)-C(a_3+b_3)$.

3. The Numerical Analysis of the Experiment

We may now evaluate the results of the experiment that was performed. We obtained the (2.1) with

$|c_+| = 0.8165$ and

$|c_-| = 0.5773$

and

$$\cos\vartheta=-0.3563. \quad \textbf{(3.1)}$$

Consequently, according to the (2.9), we have that

$A=0.8165\cos\vartheta$;

$B=0.8165\,sen\,\vartheta$;

(3.2)

$C = 0.5773\cos\vartheta$;
$D = 0.5773\, sen\vartheta$;

We may now express the probabilities, $P_+(t)$ and $P_-(t)$, given in the (2.8), for $B = \pm$ as result of quantum evolution during cognition.

First write the Hamiltonian of the subject during perception-cognition. According to the (26) of Appendix A, we have that

$$H(t) = -\frac{1}{2}(a_1 e_1 + a_2 e_2 + a_3 e_3) - \frac{1}{2}(b_1 e_1 + b_2 e_2 + b_3 e_3) \quad \textbf{(3.3)}$$

Note that the a_i (*i*= 1, 2, 3) relate the inner neurological and psychological state or condition of the human subject while the b_i relate instead the interaction of the subject with the outsider stimulus that intervenes in his perception-cognition during the posed task.

Roughly and in a preliminary way we will consider in this paper two basic cases, that one with

(1) $a_i = b_i$ ($a_i = b_i = 1$), and the other interesting case of

(2) $a_1 << b_1$, $a_2 << b_2$, and $a_3 = b_3$, fixing $a_1 = a_2 = 3$; $b_1 = b_2 = 20$; $a_3 = b_3 = 5$.

Obviously, we present here only some simple cases that instead require more careful consideration under the psychological and neurological profiles. In particular, the $b_i(t)$ relate the rate at which features of the task stimuli are integrated with human memorial representations and cognitive performance during the presentation of the task. Therefore, they must be analyzed with particular consideration. We give here only preliminary results.

The Hamiltonian in the case (1) becomes

$$H(t) = -\frac{1}{2}\begin{pmatrix} 1 & 1-i \\ 1+i & -1 \end{pmatrix} - \frac{1}{2}\begin{pmatrix} 1 & 1-i \\ 1+i & -1 \end{pmatrix}$$

while in the case (2) it is

$$H(t) = -\frac{1}{2}\begin{pmatrix} 5 & 3-3i \\ 3+3i & -5 \end{pmatrix} - \frac{1}{2}\begin{pmatrix} 5 & 20-20i \\ 20+20i & -5 \end{pmatrix}$$

The probabilities, given in the (2.8), may be now calculated in the case (1) and in the case (2), respectively.

In the case (1) we have the results given in Table 1, and in Figures 1, 2, 3.

4. The Evaluation of the Obtained Results

Let us start with examination of the results obtained in the case (1) that is

$a_i = b_i$ ($a_i = b_i = 1$).

This is the case in which we admit that it exists a strong balance between the inner psychological condition of the human subject at the moment of the submitted task and the interaction that it is established with his mind state when the task is given. From the data we deduce that both $P_+(t)$ and $P_-(t)$, at the moment of perception and cognition, start to fluctuate in time with $P_+(t)$ oscillating between a minimum value about 0.45 and a maximum value about 0.95 while $P_-(t)$ oscillates between a minimum value approaching zero and a maximum value about 0.55. As in a "quantum random walk", the mind of the subject oscillates with regularity between such different values of $P_+(t)$ and $P_-(t)$ at each time step, integrating, at each time step, the features of the task stimulus with his memorial representation and cognitive performance that is based on his mind- structure and his fixed threshold criteria. Fluctuations of $P_+(t)$ result greater of fluctuations for $P_-(t)$ at each time step with the only but fundamental exception of the time steps corresponding to maximum uncertainty for the subject .In this case there is overlap. On the basis of such regular mechanism of fluctuations in the values of probabilities and, according to the previously mentioned threshold criteria, the subject performs his final decision in a given "response time" or "reaction time", RT.

In the case (2) that is

$a_1 << b_1$, $a_2 << b_2$, and $a_3 = b_3$,

with

$a_1 = a_2 = 3$; $b_1 = b_2 = 20$; $a_3 = b_3 = 5$

the strong balance between the inner psychological condition of the human subject at the moment of the submitted task and the interaction that it is established with his mind state when this task is given, is violated and it is assumed instead that a strong unbalancing is realized with the outsider interaction greater than the inner psychological condition of the subject. From the data we deduce that we have a strong different behavior in time for both $P_+(t)$ and $P_-(t)$. We have short time steps in which $P_+(t)$ fluctuates between a minimum value about 0.30 and a maximum value about 0.95 with corresponding fluctuations for $P_-(t)$ from a minimum of about 0.05 to a maximum about 0.70.

This time dynamical regime is followed from brief time intervals in which the fluctuations of $P_+(t)$ and $P_-(t)$ are strongly reduced as well as their maximum and minimum values oscillate now between a minimum value of about 0.55 and a maximum value of 0.75 for $P_+(t)$ and a minimum value of 0.25 and a maximum value of 0.45 for $P_-(t)$. They never tend to overlap.

In addition to a basic difference in the maximum and minimum values for $P_+(t)$ and $P_-(t)$, respect to the case of balancing condition, we have that the fluctuations in the values of probabilities oscillate now irregularly, and still the regions of overlap between $P_+(t)$ and $P_-(t)$ strongly increase. The condition of strong unbalancing induces a more evident condition of uncertain in human subject decision.

In conclusion, also a preliminary evaluation of time dynamics of quantum probabilities during the cognitive process of the human subjects is able to identify some important features that characterize the mind dynamics during human cognition under the perspective of quantum mechanics. More advanced experimental studies are in progress, applied to the present theory in order to identify other important features in our mind dynamics during the perceptive-cognitive process. In particular, the irregularity of oscillations in probabilities in the case of strong unbalancing requires a careful consideration.

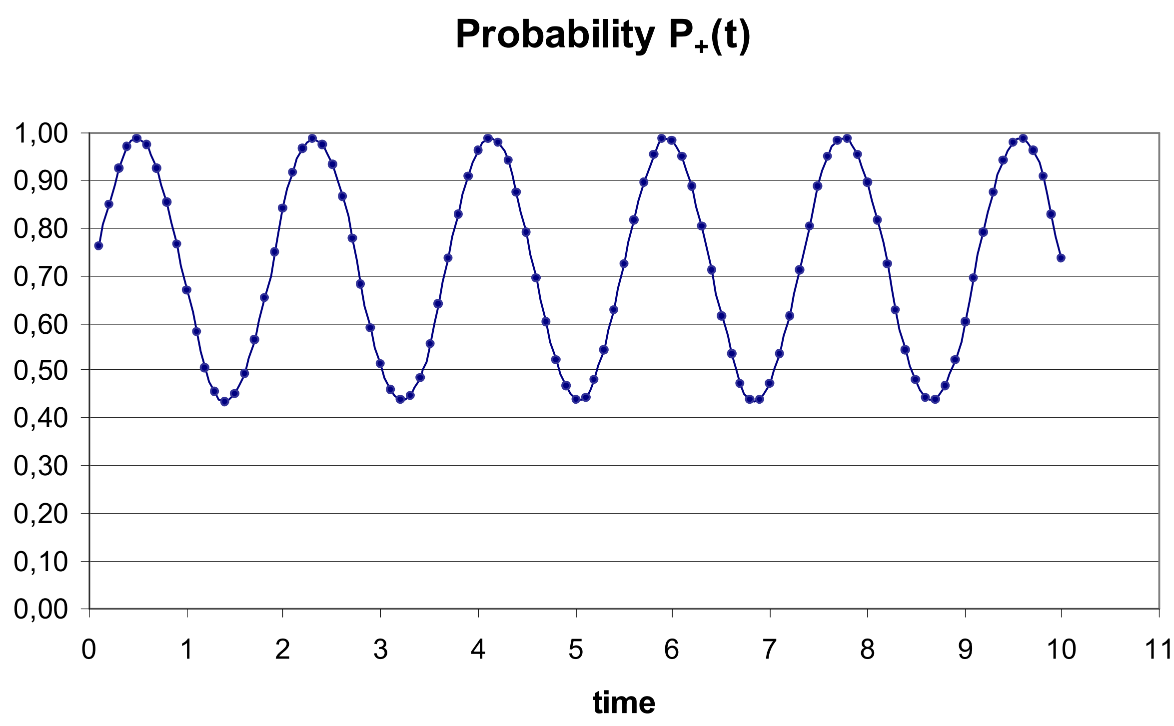


Figure 1. Please see text for explanation.

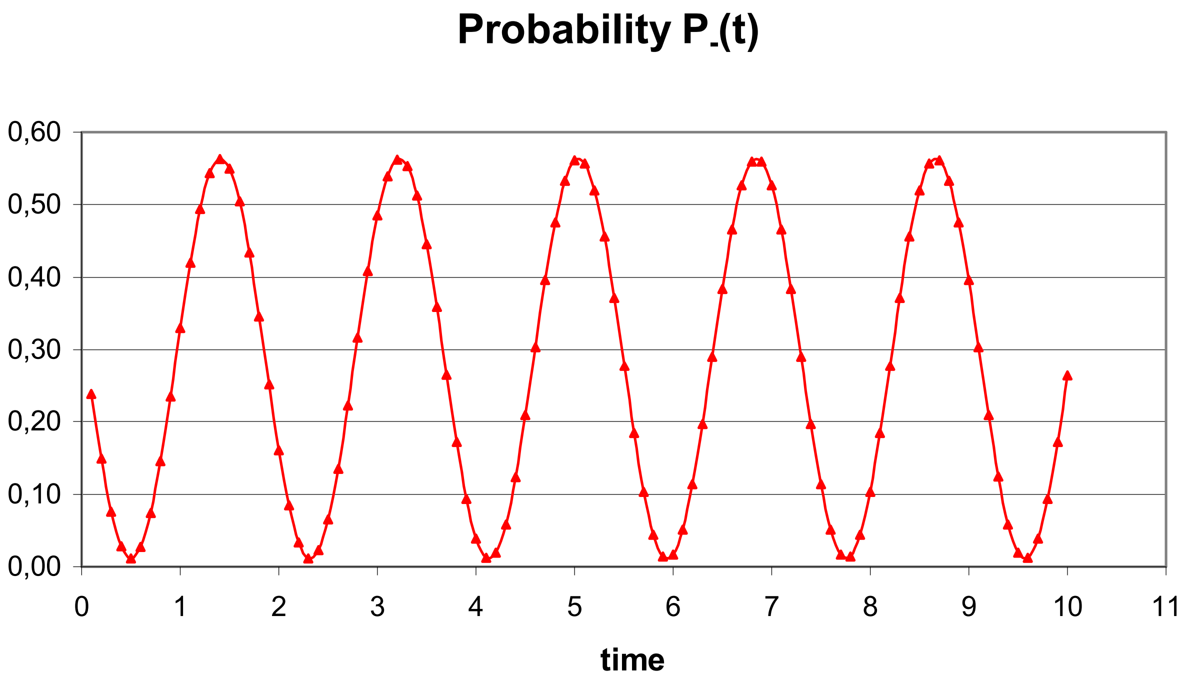


Figure 2. Please see text for explanation.

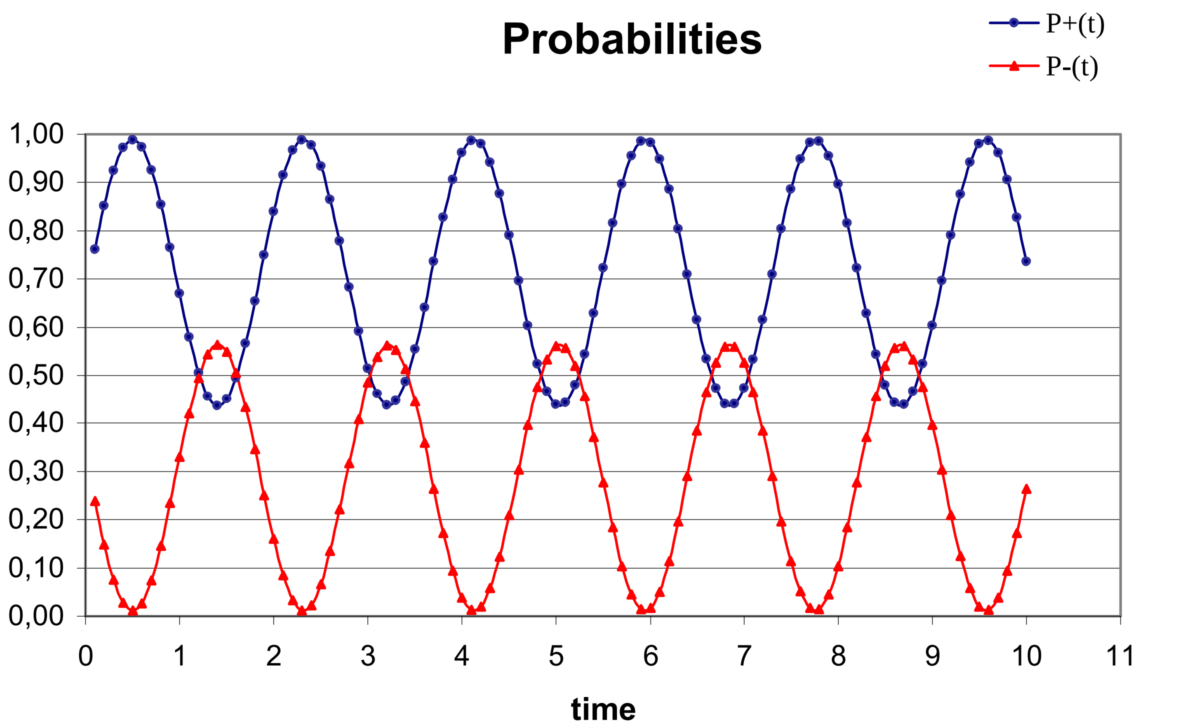
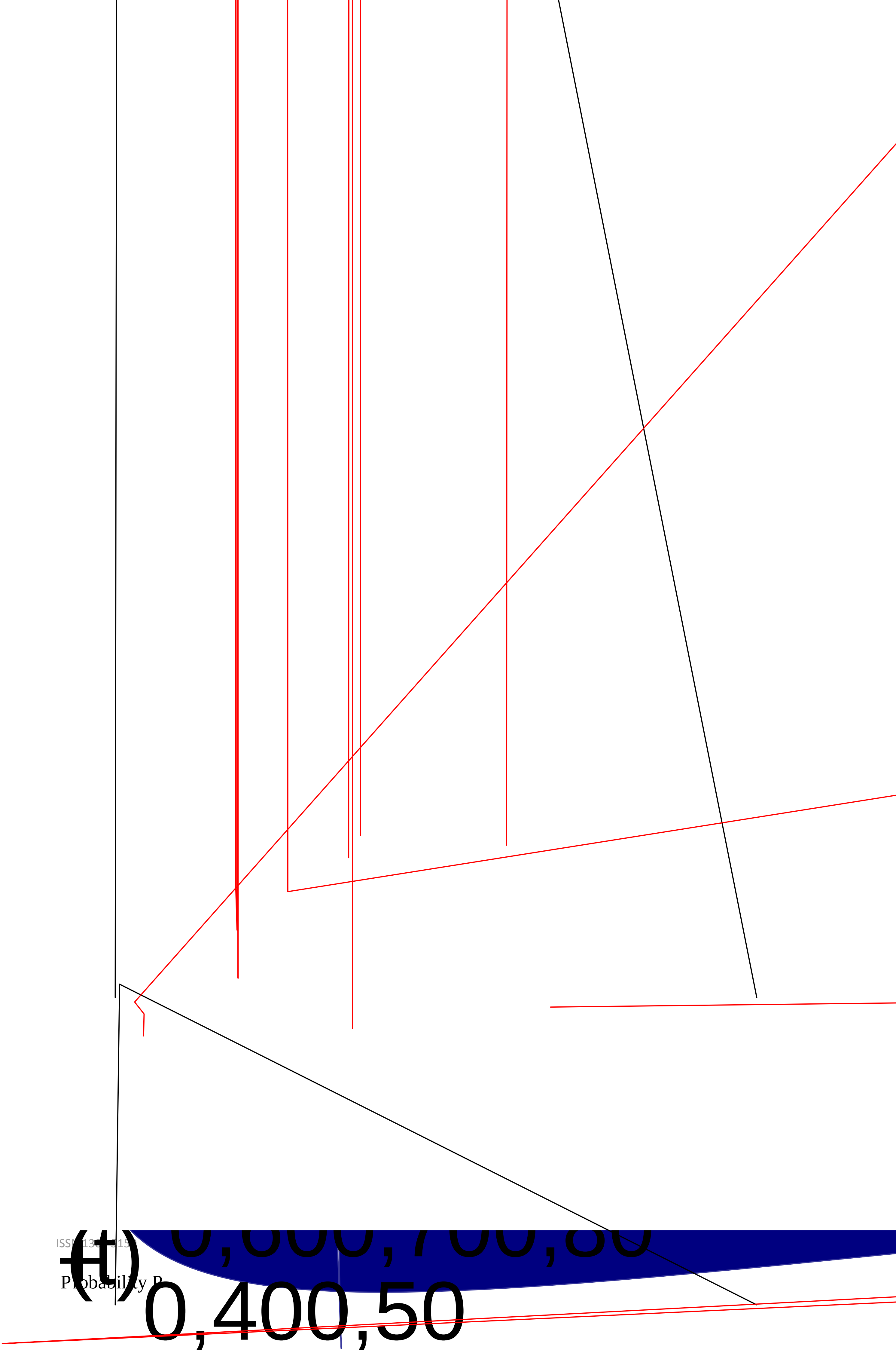


Figure 3. Please see text for explanation.


(t)
Probability P
0,400,50

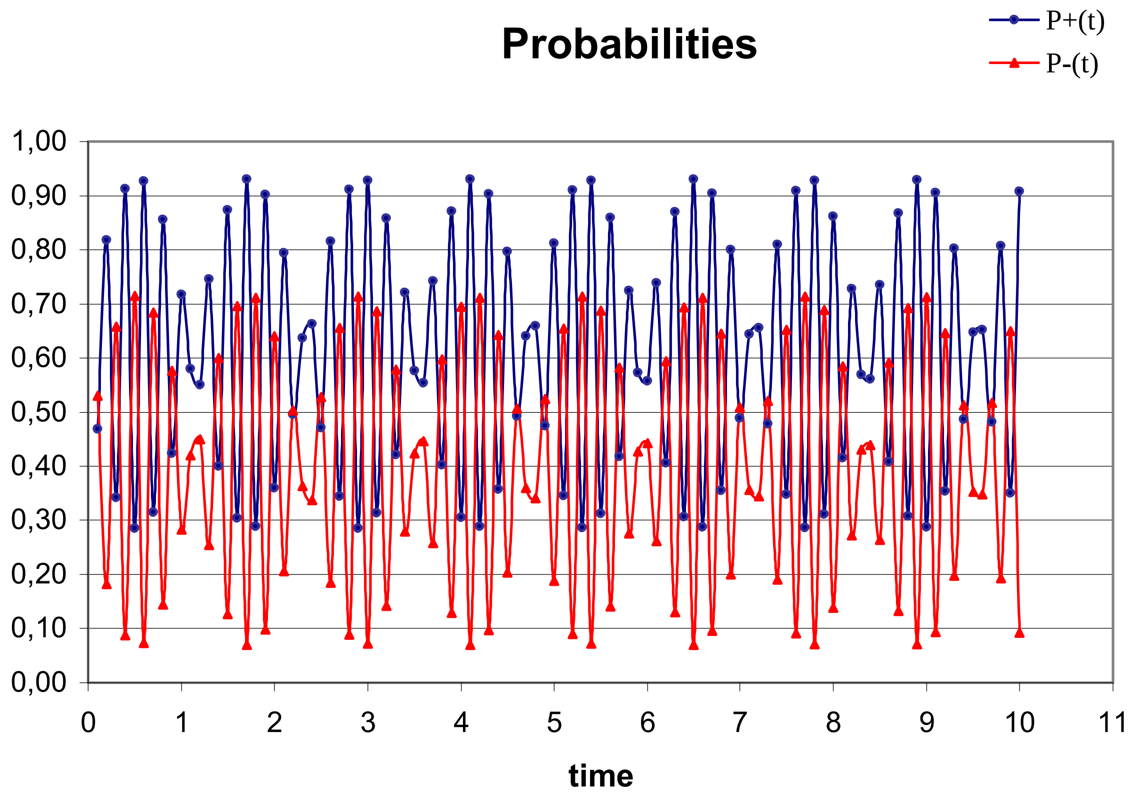


Figure 6. Please see text for explanation.

Appendix A

In abstract and formal terms we may say that we have to introduce a dynamical evolution operator U(t), time dependent, that acts on the initial state of the cognition entity. In the most simple case of the superposition given in ψ in (2.1), if we indicate such state of cognitive entity by ψ_0 to express that it is related to the initial time 0, we will write that the state of the cognitive entity at any time t, will be given by

$$\psi(t) = U(t)\psi_0 \text{ and } \psi_0 = \psi(t = 0) \tag{20}$$

The entity starts its cycle and, if left unmeasured by some cognitive measurement, remains statistically in its undifferentiated superposition state of potentialities. If, during such dynamical evolution, some cognitive measurement will start, the dynamical evolution of the superposition state will be interrupted and a final state will be selected among the ontological possibilities and yielded to be actualized on the basis of the intrinsic features of the entity and of its interaction and context. Before of all, we would examine the nature of the dynamical time evolution expressed by the (20). We have to attribute a physical meaning to the time t before the actualization will be performed owing to cognitive measurement and acting context. We will call it the time of the temporal evolution of the cognitive entity. Essentially, a Hamiltonian H must be constructed such that the evolution operator U(t), that must be unitary, gives $U(t) = e^{-iHt}$.

It is well known that, given a finite N-level quantum system described by the state ψ, its evolution is regulated according to the time dependent Schrödinger equation

$$i\hbar \frac{d\psi(t)}{dt} = H(t)\psi(t) \text{ with } \psi(0) = \psi_0 . \quad \textbf{(21)}$$

Let us introduce a model for the Hamiltonian H(t). It is the Hamiltonian of the cognitive entity. We express by H_0 the free Hamiltonian of the cognitive entity, and we consider it as a constant- internal Hamiltonian component that resumes all the basic mental, historical, social features of the considered entity. We than add to H_0 an external time varying Hamiltonian, $H_1(t)$, representing the interaction of the cognitive entity with the control fields, intending by this term all the mind and also brain influences that will act on the cognitive entity during the evolution of the initial superposition state indicated by ψ_0 as induced from the task. Thus, for the first time, we attempt here to give an unitary representation of a cognitive entity including in the time varying also the term $H_1(t)$ representing the mental contributions as well as synchronous contributions deriving from mind-brain relation when a stimulus or a task is posed to the perception-cognition of the subject. In conclusion we write the total Hamiltonian as

$$H(t) = H_0 + H_1(t) \quad \textbf{(22)}$$

so that the time evolution of the state of the cognitive entity will be given by the following Schrödinger equation

$$i\hbar \frac{d\psi(t)}{dt} = \left[H_0 + H_1(t)\right]\psi(t) \quad \textbf{(23)}$$

and $\psi(0) = \psi_0$. We have that

$$\psi(t) = U(t)\psi_0 \quad \textbf{(24)}$$

where U(t) pertains to the special group SU(N). We will write that

$$i\hbar \frac{dU(t)}{dt} = H(t)U(t) = \left[H_0 + H_1(t)\right]U(t) \quad \text{and } U(0)=I \quad \textbf{(25)}$$

Let A_1, A_2,........, A_n, ($n=N^2-1$), are skew-hermitean matrices forming a basis of Lie algebra SU(N). Assuming semiclassical approximation for external acting fields $H_1(t)$, and following the previous papers developed by Altafini (Altafini, 2003), one arrives to write the explicit expression of the Hamiltonian H(t) of the cognitive entity. It is given in the following manner

$$-iH(t) = -i\left[H_0 + H_1(t)\right] = \sum_{j=1}^{n} a_j A_j + \sum_{j=1}^{n} b_j A_j \quad \textbf{(26)}$$

where a_j and $b_j = b_j(t)$ are respectively the constant components of the free hamiltonian and the time-varying control parameters characterizing the interaction of the cognitive entity during the task and thus the human perception-cognition of the human subject. If we introduce T, the time ordering parameter, still following in detail the previous work given in (Altafini, 2003), we arrive also to express U(t) that will be given in the following manner

$$U(t) = T\exp(-i\int_0^t H(\tau)d\tau) = T\exp(-i\int_0^t (a_j + b_j(\tau))A_j d\tau) \quad \textbf{(27)}$$

that is the well known Magnus expansion (Altafini, 2003; Magnus, 1954). Locally U(t) may be expressed by exponential terms as it follows (Altafini, 2003; Magnus, 1954)

$$U(t) = \exp(\gamma_1 A_1 + \gamma_2 A_2 + \dots\dots + \gamma_n A_n) \quad \textbf{(28)}$$

on the basis of the Wein-Norman formula

$$\Xi(\gamma_1, \gamma_2, \dots\dots, \gamma_n) \begin{pmatrix} \dot{\gamma}_1 \\ \dot{\gamma}_2 \\ \dots \\ \dot{\gamma}_n \end{pmatrix} = \begin{pmatrix} a_1 + b_1 \\ a_2 + b_2 \\ \dots \\ a_n + b_n \end{pmatrix} \quad \textbf{(29)}$$

with Ξ n x n matrix, analytic in the variables γ_i. We have $\gamma_i(0) = 0$ and $\Xi(0) = I$, and thus it is invertible, and we obtain

$$\begin{pmatrix} \dot{\gamma}_1 \\ \dot{\gamma}_2 \\ \dots \\ \dot{\gamma}_n \end{pmatrix} = \Xi^{-1} \begin{pmatrix} a_1 + b_1 \\ a_2 + b_2 \\ \dots \\ a_n + b_n \end{pmatrix} \quad \textbf{(30)}$$

The present elaboration has reached now some central objectives that seem to be of considerable interest.

1) We have learned as to write explicitly the Hamiltonian of a cognitive entity.
2) Still, we have learned how to write explicitly the time evolution unitary operator U(t) regulating the dynamic time evolution of a cognitive entity in absence (b_j=0) of external influences or when mental and brain influences are present.
3) Finally, we have evidenced that ,by direct experimentation conducted on cognitive entities, we may arrive to express not only the Hamiltonian H(t) of a cognitive entity and evolution operator U(t), but we may arrive also to estimate the fundamental parameters a_j, b_j (t) that reproduce the basic features of the cognitive entity. We may also express differential equations for such parameters and variables by the γ_j introduced in the previous considered systems of differential equations (29) or (30). In brief, we have arrived to express a formalism that enables to give for the first time a satisfactory characterization of the basic cognitive and neurological features that may pertain to a cognitive entity.

In the mean while we may also see how we may render still more explicit the previously obtained results.
To reach this objective we must consider a simple case of cognitive entity based on the superposition of only two states as we considered it in the (2.1). As we outlined, we have

$$\psi = [y_1, y_2]^T \quad \text{and} \quad |y_1|^2 + |y_2|^2 = 1 \quad \textbf{(31)}$$

As previously said, we have here an SU(2) unitary transformation. Select the skew symmetric basis for SU(2). We will have that

$$e_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \; e_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad e_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \textbf{(32)}$$

Now we will consider the following matrices

$$A_j = \frac{i}{2} e_j \text{, j = 1,2,3} \quad \textbf{(33)}$$

We are now in the condition to express H(t) and U(t) in some cases of interest. In this paper we will take in consideration only the simplest case, that one of fixed and constant control parameters b_j. In subsequent papers we will take in consideration more complex and also non linear behaviors. According to (Altafini, 2003), the Hamiltonian H of the cognitive entity will become fully linear time invariant and its exponential solution will take the following form

$$e^{t(\sum_{j=1}^{3}(a_j+b_j)A_j)} = \cos(\frac{k}{2}t)I + \frac{2}{k}\mathrm{sen}(\frac{k}{2}t)\left(\sum_{j=1}^{3}(a_j+b_j)A_j\right) \quad \textbf{(34)}$$

with $k=\sqrt{(a_1+b_1)^2+(a_2+b_2)^2+(a_3+b_3)^2}$. In matrix form it will result

$$U(t)=\begin{pmatrix} \cos\frac{k}{2}t+\frac{i}{k}\mathrm{sen}\frac{k}{2}t(a_3+b_3) & \frac{1}{k}\mathrm{sen}\frac{k}{2}t[a_2+b_2+i(a_1+b_1)] \\ \frac{1}{k}\mathrm{sen}\frac{k}{2}t[-a_2-b_2+i(a_1+b_1)] & \cos\frac{k}{2}t-\frac{i}{k}\mathrm{sen}\frac{k}{2}t(a_3+b_3) \end{pmatrix} \quad \textbf{(35)}$$

and, obviously, it will result to be unimodular as required .This is the matrix representation of time evolution operator for the considered cognitive entity.

Starting with this matrix representation of time evolution operator U(t), we may deduce promptly the dynamic time evolution of the state of cognitive entity at any time t writing

$$\psi(t)=U(t)\psi_0 \quad \textbf{(36)}$$

On the general case of a dichotomous quantum variable, we are considering here that c_+ states for c_{true} and c_- for c_{false}. On this general plane, we have for ψ_0 the following expression

$$\psi_0=\begin{pmatrix} c_{true} \\ c_{false} \end{pmatrix} \quad \textbf{(37)}$$

having assumed for the true and false states the following matrix expressions

$$\varphi_{true}=\begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } \varphi_{false}=\begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \textbf{(38)}$$

Finally, one obtains the expression of the state $\psi(t)$ of the cognitive entity at any time

$$\psi(t)=\left[c_{true}\left[cos\frac{k}{2}t+\frac{i}{k}sen\frac{k}{2}t(a_3+b_3)\right]+c_{false}\left[\frac{1}{k}sen\frac{k}{2}t[(a_2+b_2)+i(a_1+b_1)]\right]\right]\varphi_{true}+$$
$$\left[c_{true}\left[\frac{1}{k}sen\frac{k}{2}t[i(a_1+b_1)-(a_2+b_2)]\right]+c_{false}\left[cos\frac{k}{2}t-\frac{i}{k}sen\frac{k}{2}t(a_3+b_3)\right]\right]\varphi_{false} \quad \textbf{(39)}$$

As consequence, the two probabilities $P_{true}(t)$ and $P_{false}(t)$ that are expected for future selection to, true or false, as consequence of cognition measurement and context influence, will be given at any time t by the following expressions

$$P_{true}(t)=(A^2+B^2)\cos^2\frac{k}{2}t+\frac{1}{k^2}\mathrm{sen}^2\frac{k}{2}t(P^2+Q^2)+\frac{\mathrm{sen}\,kt}{k}(AP+BQ)$$

and **(40)**

$$P_{false}(t) = (C^2 + D^2)\cos^2\frac{k}{2}t + \frac{1}{k^2}\,sen^2\frac{k}{2}t(S^2 + R^2) + \frac{sen\,kt}{k}(RC + DS)$$

where
A= Re c_{true}, B=Im c_{true}, C=Re c_{false}, D=Im c_{false},
P=-D(a_1+b_1)+C(a_2+b_2)-B(a_3+b_3),
Q=C(a_1+b_1)+D(a_2+b_2)+A(a_3+b_3), **(41)**
R=-B(a_1+b_1)-A(a_2+b_2)+D(a_3+b_3),
S=A(a_1+b_1)-B(a_2+b_2)-C(a_3+b_3)

As it is seen, our initial purpose to introduce abstract quantum formalism in order to describe the time dynamics of a cognitive entity has been now fully reached. By using proper experimentation we are now in the condition to analyze cognitive behavior in simple cases of control fields as well as in cases of more complex and non linear dynamical conditions. In any case the finality will be to analyze cognitive dynamics and its basic interactions by establishing with the experiments the correct behavior of the constant parameters a_j and of the time dependent functions $b_j(t)$ that regulate the time dependent behavior of the acting control fields during the dynamics of the cognitive process.

Note: For a refinement of the present calculations see also http://philpapers.org/archive/CONDM-3.1.pdf

Table 2

time	$P_+(t)$	$P_-(t)$	$P_+(t)+P_-(t)$	time	$P_+(t)$	$P_-(t)$	$P_+(t)+P_-(t)$
0.1	0.4690	0.5310	0.9999	5.1	0.3455	0.6544	0.9999
0.2	0.8179	0.1821	0.9999	5.2	0.9106	0.0893	0.9999
0.3	0.3415	0.6585	0.9999	5.3	0.2858	0.7142	0.9999
0.4	0.9130	0.0869	0.9999	5.4	0.9279	0.0720	0.9999
0.5	0.2851	0.7148	0.9999	5.5	0.3121	0.6879	0.9999
0.6	0.9267	0.0732	0.9999	5.6	0.8598	0.1402	0.9999
0.7	0.3150	0.6849	0.9999	5.7	0.4174	0.5826	0.9999
0.8	0.8553	0.1446	0.9999	5.8	0.7244	0.2755	0.9999
0.9	0.4231	0.5768	0.9999	5.9	0.5736	0.4263	0.9999
1	0.7178	0.2821	0.9999	6	0.5580	0.4420	0.9999
1.1	0.5806	0.4193	0.9999	6.1	0.7390	0.2609	0.9999
1.2	0.5510	0.4489	0.9999	6.2	0.4049	0.5951	0.9999
1.3	0.7454	0.2545	0.9999	6.3	0.8694	0.1305	0.9999
1.4	0.3994	0.6005	0.9999	6.4	0.3060	0.6939	0.9999
1.5	0.8735	0.1264	0.9999	6.5	0.9300	0.0699	0.9999
1.6	0.3035	0.6964	0.9999	6.6	0.2878	0.7122	0.9999
1.7	0.9307	0.0692	0.9999	6.7	0.9046	0.0953	0.9999
1.8	0.2889	0.7110	0.9999	6.8	0.3551	0.6449	0.9999
1.9	0.9017	0.0982	0.9999	6.9	0.8000	0.1999	0.9999
2	0.3595	0.6404	0.9999	7	0.4899	0.5100	0.9999
2.1	0.7943	0.2056	0.9999	7.1	0.6441	0.3558	0.9999
2.2	0.4965	0.5035	0.9999	7.2	0.6563	0.3437	0.9999
2.3	0.6371	0.3628	0.9999	7.3	0.4786	0.5214	0.9999
2.4	0.6632	0.3367	0.9999	7.4	0.8097	0.1902	0.9999
2.5	0.4722	0.5278	0.9999	7.5	0.3476	0.6523	0.9999
2.6	0.8152	0.1848	0.9999	7.6	0.9093	0.0906	0.9999
2.7	0.3435	0.6565	0.9999	7.7	0.2862	0.7138	0.9999
2.8	0.9118	0.0881	0.9999	7.8	0.9285	0.0715	0.9999
2.9	0.2854	0.7145	0.9999	7.9	0.3107	0.6893	0.9999
3	0.9273	0.0726	0.9999	8	0.8620	0.1380	0.9999
3.1	0.3135	0.6864	0.9999	8.1	0.4146	0.5854	0.9999
3.2	0.8576	0.1424	0.9999	8.2	0.7277	0.2722	0.9999
3.3	0.4202	0.5797	0.9999	8.3	0.5701	0.4298	0.9999
3.4	0.7211	0.2788	0.9999	8.4	0.5614	0.4385	0.9999
3.5	0.5771	0.4228	0.9999	8.5	0.7358	0.2642	0.9999
3.6	0.5545	0.4454	0.9999	8.6	0.4076	0.5923	0.9999
3.7	0.7422	0.2577	0.9999	8.7	0.8673	0.1326	0.9999
3.8	0.4021	0.5978	0.9999	8.8	0.3073	0.6926	0.9999
3.9	0.8715	0.1285	0.9999	8.9	0.9296	0.0703	0.9999
4	0.3048	0.6952	0.9999	9	0.2873	0.7127	0.9999
4.1	0.9304	0.0696	0.9999	9.1	0.9060	0.0939	0.9999
4.2	0.2883	0.7116	0.9999	9.2	0.3529	0.6470	0.9999
4.3	0.9032	0.0968	0.9999	9.3	0.8028	0.1971	0.9999
4.4	0.3573	0.6427	0.9999	9.4	0.4866	0.5133	0.9999
4.5	0.7972	0.2028	0.9999	9.5	0.6476	0.3523	0.9999
4.6	0.4932	0.5068	0.9999	9.6	0.6528	0.3472	0.9999
4.7	0.6406	0.3593	0.9999	9.7	0.4818	0.5181	0.9999
4.8	0.6597	0.3402	0.9999	9.8	0.8070	0.1930	0.9999
4.9	0.4754	0.5246	0.9999	9.9	0.3497	0.6502	0.9999
5	0.8125	0.1875	0.9999	10	0.9080	0.0919	0.9999

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