Men Who Made a New Science

Invited Article

On the Unification of Mind and Matter

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Abstract

The "hard problem" is considered vis-à-vis Gödel's work on formal systems, tensor network theory, the vector character of sensory qualities, the symmetries and phase relations of those qualities and Heisenberg's matrix formulation of quantum theory. An identity is proposed between the secondary properties of perception and (1) the hidden variables of quantum theory; (2) the internal spaces of gauge theory; and (3) the additional dimensions of M-theory.

Key Words: quantum field theory, mind/body problem, color, vision, Gödel, EPR, hidden variables, M-theory, mind-brain identity theory, tensor network theory, secondary qualities, Einstein, Bohr, Heisenberg, Pellionisz, Llinas, Churchland

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The assembling of all these elements
has been effected by century by century,
in past ages down to our own time…

Al-Kindi (ca. 800–870 CE)

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Introduction

A little over 20 years ago, when I first began to air my ideas regarding the quantum substrate of mind and brain, the topic was hardly on the map. For a long while, I took comfort in the famous dictum of Freeman Dyson, who advises us that, "For any speculation which does not at first glance look crazy, there is no hope."

Today, thanks to Sultan Tarlaci, it seems clear that the subject has attained a certain maturity and I should like to thank him for this fine honor. It seems to me that no reward can equal or surpass the appreciation of one's peers. I should also like to thank my teachers for their patient guidance and instruction.

My parents bought me a toy robot when I was a little tyke. A terror in diapers, I took it apart right away — such contrivances being beneath me — to see what made the thing go. I got as far as the motor, where I discovered coiled wire and magnets. I then read up on Faraday and Maxwell until I encountered the latter's equations and was thoroughly mystified.

Years later, I went to see the premier of *2001: A Space Odyssey.* I was quite taken with HAL, the sentient computer. It was about this time that I began to wonder how one might engineer such a device. At 16, reading Bertrand Russell, I was arrested by the problem of how the brain can perceive *color,* which seemed to be related to light — a *physical* thing — but which had long been held to reside in the mind — a *mental* thing. This proved a fortunate choice of problems; years later, I came across a remark from William James:

> The ultimate of ultimate problems, of course, in the study of the relations of thought and brain, is to understand why and how such disparate things are connected at all […] We must find the minimal mental fact whose being reposes directly on a brain-fact; and we must similarly find the minimal brain event which will have a mental counterpart at all.

Eventually it came together for me that a color vector is a "minimal mental fact" which reposes directly on a "minimal brain event," viz., a photonic state vector. Long before that idea came to light, I made an amusing blunder, however.

In my youthful explorations of physics it became pretty clear that the color of a thing didn't seem to depend on its motion in space-time — except in the important case of Doppler shifts. Other things being equal, however, the color of a thing does not change as our world spins on its axis or flies along through the interstellar regions.

Today I see, with the benefit of hindsight, that these constants in the appearances of things are instances of symmetries or invariants — and so part of Klein's rich legacy. But back then I felt stymied and so turned to neuroscience and to the logic and philosophy of science.

I soon found the seminal work by McCulloch and made a lengthy study of mathematical logic, which led to Russell & Whitehead, Tarski, Turing and Gödel. Trying to get a handle on Gödel's work quickly led me to the famous essay by Nagel & Newman, where I encountered the following passage:

> How did Gödel prove his conclusions? Up to a point, the structure of his demonstration is modeled, as he himself noted, on the reasoning involved in one of the logical antinomies known as the "Richard Paradox," first propounded by the French mathematician, Jules Richard, in 1905 […]
> The reasoning in the Richard Paradox is evidently fallacious. Its construction nevertheless suggests that it might be possible to "map" (or "mirror") metamathematical statements about a sufficiently comprehensive formal system into the system itself. If this were possible, then metamathematical statements about a system would be represented by statements within the system. *Thereby one could achieve the desirable end of getting the formal system to speak about itself — a most valuable form of self-consciousness.*[*]

That last bit fired my thinking. The brain, being a physical thing, is presumably a (quantum!) mechanical thing, and can therefore be

[*] *My emphasis*

modeled by a formal system. Moreover, our brains describe their states — they talk about themselves. They talk about objects of perception, which are represented in us as configurations of colors and sounds — which properties appear to be elemental in the sense that we cannot define them in respect of simpler entities. Years later, it hit me that, rather than try to reduce these elemental properties to simpler entities, that maybe we ought to take nature at her word and ask what sort of geometry might result if we were to alter the axiom of physical science which has it that colors and sounds exist only in the mind. What if we were to regard colors and sounds as the elements of a formal theory (**T**), where the interpretation of **T** was an EPR-complete quantum theory?

I was an undergraduate when I first began to explore these notions and spoke about them to anyone who would listen. No one understood what I was on about and I suppose they suspected I didn't, either. Fortunately for me, Douglas Hofstadter's wonderful book on *Gödel, Escher, Bach* came out about that time. A similar episode occurred a few years later; I'd published a paper on the quantum substrate of the mind and brain in 1985 and presented another in 1987. The former was greeted with the customary silence, but the latter work prompted quite a lot of interest. After my little talk, I had lunch with a bright young PhD student from California, who told me, "Yeah, you could tell — at first, everyone was wondering what planet you stepped off of; then they were all really interested."

Then, in 1989, Penrose published his first big book on these topics and the ball really began to roll, with scholarly conferences organized and fiery rhetoric detonated. A little later on, while riding a bus and working on something completely different, I had another inspiration. Years before, in the course of many happy readings of the Churchlands' works, I encountered the *tensor network theory* of Pellionisz and Llinas. I'd known for some time that Heisenberg's approach to quantum mechanics employed matrices operating on vectors. Riding the bus that day, it hit me all at once that *the matrices of tensor network theory and the matrices of QM are the same matrices and that our perceptual fields just are quantum fields.* Most of my subsequent efforts were devoted to seeing whether this inspiration could stand up to the light of day. In all this, I was guided by the ethos of simplicity admirably framed by Einstein: "When the solution is simple, God is answering." I believe that the answer to the mind/body duality is quite simple — though sweeping in its ramifications — and in what follows I have tried to be easy on the reader, mindful of the interdisciplinary nature of the work and of the real difficulties encountered in the contributing fields.

Where do we go from here? I am profoundly gratified to see that Dr. Tarlaci's pioneering efforts have been followed by other success stories, as witness the publication in the *Proceedings of the Royal Society* of Henry Stapp's ideas and by the forthcoming symposium on *Quantum Interaction,* hosted by Stanford University and the AAAI.

Although the "quantum mind" movement continues to find its detractors, their comments tend to evince only a passing acquaintance with quantum theory and/or neuroscience and strike me as the kind of rearguard actions which Kuhn discusses in his landmark work on *The Structure of Scientific Revolutions.* I think there are good reasons for thinking so; one reads, e.g., that *Nature Neuroscience* has just now published an article telling us that the "noise" in the brain — often thought to preclude quantum effects — is actually just what one might expect to find if the brain is engaged in Bayesian computations. And then, it seems as though scarcely a week goes by, lately, but one reads about the remarkable electronic properties of microtubules.

On the whole, I see good reasons for optimism.

Simplicity

Quantum field theory (QFT) enjoys an unfortunate reputation for being difficult. Without diminishing the very real challenges presented by that body of thought, it helps to bear in mind that a field is basically quite a simple thing, as Gerard 't Hooft reminds us: "a field is simply a quantity defined at every point throughout some region of space and time." Freeman Dyson helps us further:

> This is the characteristic mathe- matical

> property of a classical field: it is an undefined something which exists throughout a volume of space and which is described by sets of numbers, each set denoting the field strength and direction at a single point in the space.

Dyson also tells us, with the simplicity of genius, that "There is nothing else except these fields: the whole of the material universe is built of them."

What has QFT to do with consciousness, though? In the helpful case of visual perception we often refer to the visual *field,* where colors would seem to be akin to Dyson's "undefined something which exists throughout a volume of space."

In this wise it is intriguing to reflect upon a terse observation from Wittgenstein:

> A speck in the visual field, though it need not be red must have some color; it is, so to speak, surrounded by color-space. Notes must have some pitch, objects of the sense of touch some degree of hardness, and so on.

On a mind/brain identity theory, such as we find in Bohm, Chalmers, Feigl, Lockwood and Russell, it is tempting to suppose that our perceptual fields just are electromagnetic fields. Abdus Salam helps us along here, reminding us that "all chemical binding is electromagnetic in origin, and so are all phenomena of nerve impulses."

If we should suppose, as would seem altogether plausible, that conscious processes just are phenomena of nerve impulses, then it seems to follow as a ready consequence that those conscious processes are electromagnetic in origin. Can we put flesh on the bones of this conjecture?

Dyson wrote that a field is "is an undefined something which exists throughout a volume of space." Maxwell tells us that color is undefined, in the sense that colors are so very simple, they are incapable of analysis:

> When a beam of light falls on the human eye, certain sensations are produced, from which the possessor of that organ judges of the color and luminance of the light. Now, though everyone experiences these sensations and though they are the foundation of all the phenomena of sight, yet, on account of their absolute simplicity, they are incapable of analysis, and can never become in themselves objects of thought. If we attempt to discover them, we must do so by artificial means and our reasonings on them must be guided by some theory.

Russell and Whitehead back up Maxwell: "Thus 'this is red,' 'this is earlier than that,' are atomic propositions."

So far as colors existing through- out a volume of space goes, we are on safe ground, thanks to Riemann, who, at the beginning of his famous habilitation lecture on the foundations of geometry, points out that colors and the positions of objects both define manifolds:

> So few and far between are the occasions for forming notions whose specialization make up a continuous manifold, that the only simple notions whose specialization form a multiply extended manifold are the positions of perceived objects and colors. More frequent occasions for the creation and development of these notions occur first in the higher mathematics.
>
> Definite portions of a manifold, distinguished by a mark or a boundary, are called Quanta [...].

Here is Hermann Weyl to help us out a bit more on manifolds: "The characteristic of an n-dimensional manifold is that each of the elements composing it (in our examples, single points, conditions of a gas, colors, tones) may be specified by the giving of n quantities, the 'co-ordinates,' which are continuous functions within the manifold."

Dyson wrote that that fields are "described by sets of numbers, each set denoting the field strength and direction at a single point in the space." That sounds like a vector and it just so happens that colors behave like vectors, too, as Grassmann, Maxwell, Schrödinger, Weyl and Feynman all tell us — and as the technology behind our color TVs and computer monitors makes plain.*

* The seminal works by Grassmann, Maxwell and Schrödinger

Appealing to Feigl's formulation of mind/brain identity theory, we might say that configurations of colors represent other objects in the natural world, as when we describe an apple as a red, roundish thing. Already we encounter a difficulty, however, for we want to know where the redness comes in. Schrödinger's equation is often said to contain, in principle, all that can be known about a physical system. Schrödinger disagreed, however:

> If you ask a physicist what is his idea of yellow light, he will tell you that it is transversal electromagnetic waves of wavelength in the neighborhood of 590 millimicrons. If you ask him: But where does yellow comes in? he will say: In my picture not at all, but these kinds of vibrations, when they hit the retina of a healthy eye, give the person whose eye it is the sensation of yellow.

How did we get here? The spectrum of colors is arrayed, in a predictable, reliable manner, with the spectrum of wavelengths found in visible light. And even quite intelligent people will hasten to tell you that color just *is* wavelength — not knowing that colors are vectors, whereas wavelengths are scalars, having only magnitude.[†] Weyl reminds us of the essential facts:

> To monochromatic light corres- ponds in the acoustic domain the simple tone. Out of different kinds of monochromatic light composite light may be mixed, just as tones combine to a composite sound. This takes place by superposing simple oscillations of different frequency with definite intensities.

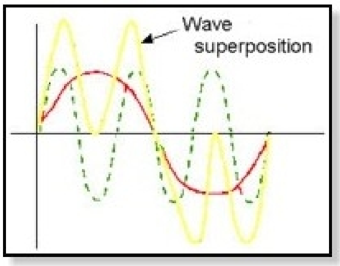

Figure 1. Wave superposition

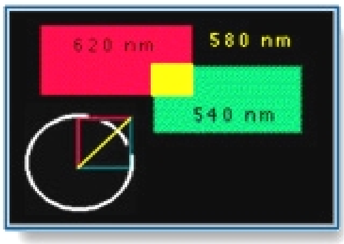

Figure 2. Vector superposition; notice that the normalized yellow vector is a precise metamer for yellow at 580 nm.

We all know these things — and yet these familiar, characteristic *(eigen)* color vectors and tones (predictably related to "simple oscillations of different frequency with definite intensities") these colors and tones are not yet properly incorporated into the body of physical theory. So, again: How did we come to this pass?

History

The answer can be found in the classic works of Burtt, Duhem, and Lovejoy, but Weyl puts the matter succinctly:

> The idea of the merely subjective, immanent nature of sense qualities, as we have seen, always occurred in history woven together with the scientific doctrine about the real generation of visual and other sense perceptions […] Locke's standpoint in distinguishing primary and secondary qualities corresponds to the physics of Galileo, Newton, and Huyghens; for here all occurrences in the world are constructed as intuitively conceived motions of particles in intuitive space. Hence an

can be found in the superb collection edited by MacAdam.

† Sounds also come to us in spectra, with remarkable relations between tone and number, already known to Pythagoras — though still awaiting a good explanation.

> absolute Euclidean space is needed as a standing medium into which the orbits of motion are traced. One can hardly go amiss by maintaining that the philosophical doctrine was abstracted from or developed in close connection with the rise of this physics.

Austen Clark elaborates on the essential situation wherein we find ourselves:

> The world as described by natural science has no obvious place for colors, tastes, or smells. Problems with sensory qualities have been philosophically and scientifically troublesome since ancient times, and in modern form at least since Galileo in 1623 identified some sensory qualities as characterizing nothing real in the objects them- selves […]
>
> The qualities of size, figure (or shape), number and motion are for Galileo the only real properties of objects. All other qualities revealed in sense perception — colors, tastes, odors, sounds, and so on — exist only in the sensitive body, and do not qualify anything in the objects themselves. They are the effects of the primary qualities of things on the senses. Without the living animal sensing such things, these 'second- ary' qualities (to use the term introduced by Locke) would not exist.
> Much of modern philosophy has devolved from this fateful distinction. While it was undoubtedly helpful to the physical sciences to make the mind into a sort of dustbin into which one could sweep the troublesome sensory qualities, this stratagem created difficulties for later attempts to arrive at some scientific understanding of the mind. In particular, the strategy cannot be reapplied when one goes on to explain sensation and perception. If physics cannot explain secondary qualities, then it seems that any science that can explain secondary qualities must appeal to explanatory principles distinct from those of physics. Thus are born various dualisms.

Galileo and Newton carried the day, along with their philosophical admirers, viz., Locke, Hobbes and Descartes — over the objections of Leibniz and also Hume — for the simple reason that their new science worked, and marvelously so. And one might well ask, if the new physics was really so flawed, why does it continue to work so well? (That is, until we try to frame a science of perception?)

Completeness

Thoughtful persons will see that we have encountered the former question at the foundations of quantum theory, in the Einstein-Bohr debates, in EPR and in the literature that followed.

Now, a notoriously intractable issue in contemporary thought on the mind/body problem concerns what Chalmers has famously dubbed the “hard problem,” viz., the question of why some brain states have perceptual states attached to them. Others, whom I suspect betray a guilty conscience, have attempted to wave away the issue, dismissing it as mere talk regarding “the redness of red.” Often missing from these discussions is any reference to the scientific literature concerning color. This is a bit odd, given that such illustrious persons as Newton, Young, Helmholtz, Grassmann, Maxwell, Schrodinger, Weyl, Einstein and Feynman have devoted serious thought to the nature of color.

As noted above, Schrödinger's wave function is often said to contain, in principle, all that can be known about a physical system. Einstein disagreed:

> one arrives at very implausible theoretical conceptions, if one attempts to maintain the thesis that the statistical quantum theory is in principle capable of producing a complete description of an individual physical system.

The seminal work of EPR raised the possibility that QM is *incomplete,* in the sense that not all “elements of reality” are incorporated into the body of quantum theory. Those missing elements have come to be known as “hidden variables,” and I am happy to see that these variables are once again on the front burner of theoretical physics, thanks to Hartle, Holland, 't Hooft, Peres and Smolin, among others. I should like to pause here to pay tribute to the path finding work of David Bohm and J. S. Bell, who, nearly alone, kept this question alive for 60 years, when the foundations of quantum theory languished in a moribund state, thanks to the "brainwashing" which, as Gell-Mann tells us, Bohr and Heisenberg induced in a generation of physicists.

The proposal that I put forth over 20 years ago is that the "secondary qualities" of color and sound and so forth, long ago banished to the misty realm of the mind, just *are* EPR's missing elements of reality — variables only hidden in plain sight, their true nature obscured by 300 years of dogma. As scientists, we take in this doctrine with our mothers' milk; it forms part of what "everybody knows," and so constitutes a brainwashing more thorough than anything Heisenberg and Bohr ever devised — and this despite the fact (picked up at the time by Hume and Leibniz) that this doctrine lies at the very crux of the scientific worldview.

I don't want to be too hard on Bohr and Heisenberg, whose place in the pantheon of physics is, of course, beyond question and whose thoughts on the questions at hand I will now exploit.

As David Bohm informs us in the introduction to his congenial book on *Quantum Theory,* "Bohr suggests that thought involves such small amounts of energy that quantum-theoretical limita- tions play an essential role in determining its character." Bohr also wrote to the effect that nothing is so fundamental to the worldview of quantum theory as the demolition of the notion that physics has to do with what is *out there* — i.e., the Newtonian ideal of a world where a detached observer presents us with no problems.

Henry Stapp has put Heisenberg's ideas about QM and the observer problem to good use, arguing that quantum theory has essentially to do with our *knowledge* of physical states and systems. He has made a bold and intriguing conjecture — drawing on William James's ideas about the selective nature of attention — that it is our conscious attention that is behind the Quantum Zeno Effect (QZE). For my part, I am tend to agree with J.S. Bell, who thought that the observer ought not to be "ontologically privileged." I expect that the QZE is due to nonlocal effects — but then, it may well not be a case of either/or: If consciousness does consist of quantum field processes, those processes might then have nonlocal effects, producing the QZE. I do not know, of course — I don't think anyone does at this juncture — but it seems a fascinating possibility.

I will return to Heisenberg later on, but would like to point out that five other prominent physicists also devoted serious thought to mind and matter, namely Einstein, Pauli, Schrodinger, Weyl and Wigner. Aside from Pauli, whose thoughts on these issues have only recently come to light, I have remarked in another place upon the contributions of these men, and so I only touch upon them here. Wigner, e.g., wrote that physics would have to be transformed in order to account for consciousness. Pauli, in a recently translated series of letters to Carl Jung, looked forward to a unitary description of mind and matter, informed by quantum theory.

Logic

Einstein, in his "Remarks on Bertrand Russell's Theory of Knowledge," quotes Russell in this connection, going to the heart of the matter:

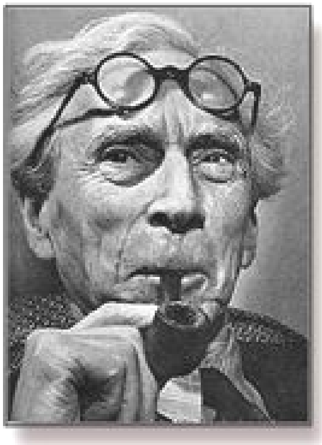

Figure 3. Russell

> We all start from 'naive realism,' i.e., the doctrine that things are what they seem. We think that grass is green, that stones are hard, and that snow is cold. But physics assures us that the greenness of grass, the hardness of stones, and the coldness of snow, are not the greenness, hardness, and coldness that we know in

> our own experience, but something very different. The observer, when he seems to himself to be observing a stone, is really, if physics is to be believed, observing the effects of the stone upon himself. Thus science seems to be at war with itself: when it means to be most objective, it finds itself plunged into subjectivity against its will.

Einstein replies:

> Apart from their masterful formula- tion these lines say something which had never previously occurred to me. For, superficially considered, the mode of thought of Berkeley and Hume seems to stand in contrast to the mode of thought in the natural sciences. However, Russell's just cited remark uncovers a connection: If Berkeley relies upon the fact that we do not directly grasp the "things" of the external world through our senses, but that only events causally connected with the presence of "things" reach our sense-organs, then this is a consideration which gets its persuasive character from our confidence in the physical mode of thought.

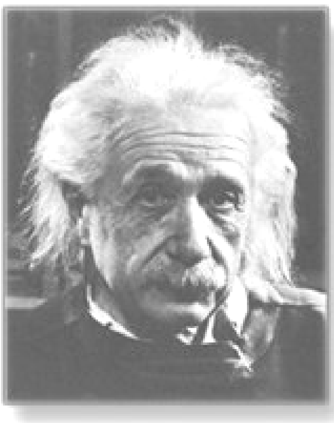

Figure 4. Einstein

Now, our usual "confidence in the physical mode of thought" stems from its conceptual beauty, logical clarity, predictive power and practical utility. Nonetheless, we are left with public, reproducible and reliable facts of greenness, hardness, and coldness.

As noted earlier, these properties seem to be elemental on account of their extreme simplicity. Let us take a page from Gödel, then, and frame a formal theory **T**, of a very general nature, constrained only by the laws of simple logic.

Take for the elements of **T** both the primary qualities and secondary qualities of sensory perception. It then follows as a consequence of simple logic that, no matter how complex this essentially mechanical thing we call **T** may be — it may have all the complexity of a human brain — **T** will be unable to define its elements, for the simple reason that its elements would not then be elements and we would have a simple contradiction.

In some such fashion, perhaps, are we unable to define the elements of our perceptions, those self-same elemental sensory qualities.

Can we find sensible places at the foundations of physical theory where additional variables might be introduced without damaging the structure above — or, better yet, where additional variables might put that edifice on a more secure footing? I have already suggested that hidden variables hold out that promise — for colors and sounds are surely simple things, as Maxwell, Wittgenstein, Russell and Whitehead have argued. Being so simple, they might justly be regarded as elemental. Given that these properties are evidently part of reality, they might then be thought of as EPR's "elements of reality," which have, again, yet to be explicitly incorporated into the body of quantum theory. I say "explicitly incorporated" because we already commonly ascribe these properties, in everyday speech, to various forms of EM energy — as when we speak of *warm* sunshine, *red* light and so forth. Indeed, these properties are typically the most salient aspects of light. Feynman, in his little book on *QED,* writes about blue photons and we all know what he means — until we start to think about it.

Symmetry

The move sketched in just now will strike some as a little outlandish; nonetheless, we already have in hand the kind of theory suggested above; the primary qualities of extension in space and duration in time are already part of the foundations of physics and are encoded in the metric of general relativity (GR).

Einstein showed us how to derive

gravitation from the metric tensor — and here the primary quality of mass also enters the theory. Minkowski, who was Einstein's teacher, said that "relativity" was a misnomer, for it was really a theory of *invariants,* given that relativity flows from the invariance of the speed of light for all observers. Today we call these kinds of invariants by the more suggestive name of *symmetries* — and these symmetries are of truly fundamental importance. Nobel laureate Steven Weinberg has stated the case with admirable precision: "Today, it is increasingly clear that the symmetry group of nature is the deepest thing that we understand about nature."

As will be seen, group theory also holds sway over the realm of colors and sounds.

Weyl did for electromagnetism (EM) what Einstein did for gravity. Kaluza demonstrated how, by adding an additional spatial dimension, both gravity and Weyl's EM could flow from the same metric. Weyl got off to a bit of a false start, as Einstein pointed out, thinking that length (or "gauge") was the quantity in question. A little later on, it was pointed out that the *phase* of the wave function was the quantity in question, but "gauge" stuck.

How might we coordinate the secondary properties with space-time? I have already suggested that these sensory properties are the "hidden" variables (HVs) of quantum theory. This whole realm of discourse continues to be fraught with controversy, however — the quite recent work by Hartle, Holland, Smolin, Peres, 't Hooft et al. notwithstanding. What other options do we have?

Let us see whether we can tighten up just what we need to do so far coordinating the secondary qualities with physical theory. Here is an excerpt from Lockwood's wonderful book, *Mind, Brain & Quantum;* Lockwood and I arrived at many of the same conclusions and by similar routes — and wholly independently of one another (private communication).

> Take some range of phenomenal qualities. Assume that these qualities can be arranged according to some abstract n-dimensional space, in a way that is faithful to their perceived similarities and degrees of similarity [...] Then my Russellian proposal is that there exists, within the brain, some physical system, the states of which can be arranged in some n-dimensional state space [...] And the two states are to be equated with each other: the phenomenal qualities are identical with the states of the corresponding physical system.

This is precisely right, I think; surprisingly, the mathematics of *fiber bundle theory* would seem to offer us just what we need in the way of state spaces, as the following excerpt from Atiyah suggests (where R4 is the familiar four-dimensional space-time):

> We shall now recall the data of a classical theory as understood by physicists and then reinterpret them in geometrical form.
> Geometrically or mechanically we can interpret this data as follows. Imagine a structured particle, that is a particle which has a location at a point x of R4 and an internal structure, or set of states, labeled by elements g of G.

Gauge theory	**General relativity**
Gauge transformations	Co-ordinate transformations
Gauge group	Group of all co-ordinate transformations
Gauge potential, A_μ	Connection coefficient, $\Gamma^\kappa_{\mu\nu}$
Field strength, $G_{\mu\nu}$	Curvature tensor, $R^\kappa_{\lambda\mu\nu}$
Bianchi identity:	Bianchi identity:
$\sum_{\rho\mu\nu \text{ cyclic}} D_\rho G_{\mu\nu} = 0$	$\sum_{\rho\mu\nu \text{ cyclic}} D_\rho R^\kappa_{\lambda\mu\nu} = 0$

Is the parallel between the state spaces of Lockwood and those of Atiyah more than suggestive? Today, all of particle theory is governed by gauge theory, which attaches an "internal space" to each elementary particle — a space governed by a mathematics precisely analogous to that found in relativity. The symmetries of this internal space mirror those found in space-time and these internal symmetries help guide the evolution of the particle's wave function.

Ryder's brilliantly lucid introduction to QFT provides us with a handy table that summarizes this correspondence.

The formulae above are a bit daunting for some; let us attend Cao, who makes the essential points in an easy-going manner:

> The internal space defined at each space-time point is called a fiber, and the union of this internal space with space-time is called fiber-bundle space. Then we find that the local gauge symmetries remove the 'flatness' of the fiber-bundle space since we assume that the internal space directions of a physical system at different space-times points are different.
>
> So the local gauge symmetry also requires the introduction of gauge potentials, which are responsible for the gauge interactions, to connect internal directions at different space- time points. We also find that the role the gauge potentials play in fiber-bundle space in gauge theory is exactly same as the role the affine connection plays in curved space- time in general relativity.

Curiously, colors and sounds also exhibit these fundamental symmetries. Thus, other things being equal, colors and sounds are symmetric or invariant under such physically important operations as translations, rotations and reflections. The celebrated theorem of Emmy Noether relates these kinds of symmetries to the conservation of matter, charge and angular momentum. Ramond makes this point explicit in his classic introduction to field theory:

> It is a most beautiful and awe-inspiring fact that all the fundamental laws of Classical Physics can be understood in terms of one mathematical construct called the Action. It yields the classical equations of motion, and analysis of its invariances leads to quantities conserved in the course of the classical motion. In addition, as Dirac and Feynman have shown, the Action acquires its full importance in Quantum Physics.

Here is Weinberg again to let us in on the joke: "Furthermore, and now this is the point, this is the punch line, the symmetries determine the action. This action, this form of the dynamics, is the only one consistent with these symmetries."

In brief, what I would like to suggest here, above all else, is this: In the observable symmetries of color and sound, we have a route leading from our direct experience of these traditionally "mental" properties straight into the heart of contemporary physical theory.

Colors and sounds also exhibit well known relations to the phases of the waves with which they are associated. Moreover, color vectors and sound vectors behave like the elements of a group under addition: If one makes the physically intuitive choice of assigning the inverse of **x** (for any color or sound) to –**x**, where –**x** is just **x**, but 180° out of phase, and where the zero elements are just darkness and silence — or, no light and no sound. I am rather pleased to have figured this out, as it seems to have escaped Schrödinger, judging from the handful of scientific papers he wrote about color.

If we then assign red, green and blue (or some other triple) to the principal axes of a unit sphere, we can then generate all the other colors by simple vector addition. The inverse of **x** is, again, just **x**, but pointing in the opposite direction. Of course, colors come in different magnitudes, and so we need to weight the vectors.

Projection

Let us revisit Weyl in a moment, where he points out that composite colors and tones are made "by superposing simple oscillations of different frequency with definite intensities." First, here is Dirac on the subject of superposition in quantum theory:

> When a state is formed by the superposition of two other states, it will have properties that are in some vague way intermediate between those of the original states and that approach more or less closely to those of either of them according to the greater or less 'weight' attached to this state in the superposition process. The new state is

completely defined by the two original states when their relative weights in the superposition process are known, together with a certain phase difference, the exact meaning of weights and phases being provided in the general case by the mathematical theory.

A little reflection should serve to persuade one that Dirac's remarks apply equally will to those color states which are predictably concomitant with photon states. In manufacturing color TV screens, millions of colors can be produced by superposing the light from red, green and blue pixels in fixed amounts or "weights." We also know that any light of a given color will be brightened or darkened according to whether the waves bearing that color are in phase or out of phase.

Let us now pay special attention to Dirac's comment concerning how, when one quantum state arises from the superposition of two other states, it will exhibit properties "that are in some vague way intermediate between those of the original states." What might we make of this "vague" assertion?

Weyl clarifies matters for us again, pointing out that the "betweenness" of the intermediate states can be readily modeled by an axiom from projective geometry: "Thus the colors with their various qualities and intensities fulfill the axioms of vector geometry if addition is interpreted as mixing; consequently, projective geometry applies to the color qualities."

This is an important point and so let us linger a moment or two over Weyl's reasoning.

Figure 5. Weyl

Mathematics has introduced the name isomorphic representation for the relation which according to Helmholtz exists between objects and their signs. I should like to carry out the precise explanation of this notion between the points of the projective plane and the color qualities On the one side, we have a manifold $\Sigma\Sigma_1$ of objects — the points of a convex section of the projective plane — which are bound up with one another by certain fundamental relations R, R', ...; here, besides the continuous connection of the points, it is only the one fundamental relation: 'The point C lies on the segment AB'. In projective geometry no notions occur except such as are defined on this basis. On the other side, there is given a second system $\Sigma\Sigma_2$ of objects — the manifold of colors — within which certain relations R, R', [...] prevail which shall be associated with those of the first domain by equal names, although of course they have entirely different intuitive content. Besides the continuous connection, it is here the funda- mental relation: 'C arises by a mixture from A and B'; let us therefore express it somewhat strangely by the same words we used in projective geometry: 'The color C lies on the segment joining the colors A and B.' If now the elements of the second system $\Sigma\Sigma_2$ are made to correspond to the elements of the first system $\Sigma\Sigma_1$ in such a way, that to elements in $\Sigma\Sigma_1$ for which the relation R, or R' […] holds, there always correspond elements in $\Sigma\Sigma_2$ for which the homonymous relation is satisfied, then the two domains of objects are isomorphically represented on one another. In this sense the projective plane and the color continuum are isomorphic with one another. Every theorem which is correct in the one system $\Sigma\Sigma_1$ is transferred unchanged to the other $\Sigma\Sigma_2$. A science can never determine its subject matter except up to an isomorphic representation. The idea of isomorphism indicates the self-understood, insurmountable barrier of

> knowledge. It follows that toward the "nature" of its objects science maintains complete indifference. Thus, e.g., what distinguishes the colors from the points of the projective plane one can only know in immediate alive intuition.

In brief, then, it seems as though spectral colors respect the laws of projective vector geometry. Following Maxwell's reasoning and common usage, we can map color vectors to a sphere in a manner which recapitulates their group behavior under superposition, where red, green and blue (or some other triplet of "orthogonal" colors) can be used to designate the principal axes.

This is all rather suggestive, I think, given the prominence of projective spaces in quantum theory, Kaluza-Klein theory, M-theory and the projective relativity of Einstein-Cartan. It is also interesting to note that, as with the vectors in quantum theory, color vectors at any given point of space-time are jointly exclusive and mutually exhaustive — one of the principal reasons why vectors are so useful in quantum theory, as Hughes tell us. That is, a "point" (patch) in space can not be red and green at the same time. And then, the dual language of forms would seem to allow for colored patches — such as are directly observed — while also reminding us that projective geometry provided us with the first instance of those "dualities" that are so prominent in string/M-theory.

Then, too, it seems fairly clear that visual space also has a projective character, once one considers that parallel railroad tracks appear to us as converging in the distance. Poincaré wrote that "Perceptual space is only an image of geometric space, an image altered in shape by a sort of perspective." I would like to suggest that Poincaré had it turned around somewhat, it being more accurate to say that the usual Euclidean space is a special case of projective space, as Kline makes explicit:

> it became possible to affirm that projective geometry is indeed logically prior to Euclidean geometry and that the latter can be built up as a special case. Both Klein and Cayley showed that the basic non-Euclidean geometries developed by Lobachev- sky and Bolyai and the elliptic non-Euclidean geometry created by Riemann can also be derived as special cases of projective geometry. No wonder that Cayley exclaimed, "Projective geometry is all geo- metry."

What I am trying to suggest is that the projective geometry of perceptual space is the true geometry of the world and that the world only appears flat and Euclidean to us because we typically experience it at speeds far below that of light and in fairly weak gravitational fields. On this view, projective relativity is a better fit to the much-rumored real world — and this is, again, quite suggestive, given the prominence of projective geometry in Kaluza-Klein theory and its descendants in string/M-theory. An adequate exploration of these connections would take us too far a field, however, and so must wait for another day. For now, I will simply remind the reader that, on the one hand, the additional dimensions of Kaluza-Klein theory and its descendants await a real-world interpretation. While on the other hand, the secondary properties of perception respect important symmetries and phase relations and also fiber over ("sit over") their respective spaces.

Matrix

I said I would return to Heisenberg, whose matrix mechanics rivaled Schrödinger's wave mechanics until Dirac demonstrated that the two formulations were fairly equivalent. In QM, a physical state is assigned a vector on a unit sphere in Hilbert space. Operators then rotate the vector, carrying it to a new state. And this is interesting, because colors can also be mapped to a unit sphere, as previously discussed, and in a manner faithful to their group behavior under superposition.

Heisenberg's formulation of QM would then seem to perform quite as well in describing the behavior of color, a traditionally mental object, where every physical operation which changes the color of a thing corresponds to a rotation of its color vector in color space.

Thus, for example, in an operator corresponding to the Doppler Effect, a red-shift would rotate the photonic state vector in Hilbert space *as well as its concomitant color vector,* by a fixed number of degrees and in a certain direction — doing so predictably, reliably, and quantifiably.

The picture which emerges portrays color space as a subspace of an EPR-complete Hilbert space.

Now, since a red-shift (blue-shift) corresponds to a lowering (raising) of the *energy* of the photon, we have a ready correspondence with the Hamiltonian (energy) operator, which then ought to lead to an EPR-complete Lagrangian, which then ought to embody what might be called secondary symmetries. Noether's theorem is a helpful guide, here: Energy, being a conserved quantity, gives us a constant frequency, by the fundamental relation:

$$E = h\nu$$

The frequency being constant, we fully expect the color to be invariant. Another basic equation tells us that the energy operator is the Hamiltonian, so a change in energy due to the Doppler Effect (or to a gravitational field) rotates the photonic state vector in a predictable way, yielding a predictable change of color.

$$E Y = H Y$$

Moving to the Lagrangian picture, the conservation laws lead us along a natural path to action equations and variational principles, as when we consider that, for constant E, a photon is stationary in color space — i.e., its vector continues to point in the same direction in color space. Notice also that, with color vectors, matrices and tensor operators in hand, we can plausibly avail ourselves of the tools of Riemannian geometry. So it seems we have come full circle with respect to my youthful blunder; the joke is on me, since it now seems fairly clear that the symmetries of the secondary properties lead us to the equations of motion.

We also immediately recover the familiar observable fact that, other things being equal, the secondary properties of things do not change — they are invariant or symmetric — as the earth rotates on its axis and the solar system goes flying along through space-time. Colors, sounds, and so forth are, then, symmetric under translations and rotations (and, it would seem, reflections) in space-time. Similarly, those operators which *do* change color vectors look like matrix-valued tensors, since these operators must preserve the relevant symmetries.

If I am not much mistaken, then, we seem to have here the germ of a general correspondence principle between what might be called a *primary* and a *secondary* mechanics, akin to Bohr's familiar correspondence relations between classical and quantum mechanics. Again, notice that we already have in hand the kinds of vectors and matrices in question; they are employed in the manufacture of color TVs and stereo systems. These working artifacts provide, moreover, a substantial sort of proof of the notions alluded to herein.

It is quite intriguing to note, again, in analogy with the pixels on a TV screen or computer monitor, that color space appears to fiber over ("sit over") the manifold of visual perception, where a copy of color space is assigned to each point of visual space-time. The analogy falters, for what we really need is a picture wherein, to paraphrase Wittgenstein, every pixel may be of any color. For those unfamiliar with Greene's entertaining book on *The Elegant Universe,* here is an illustration of the fiber-bundle concept.

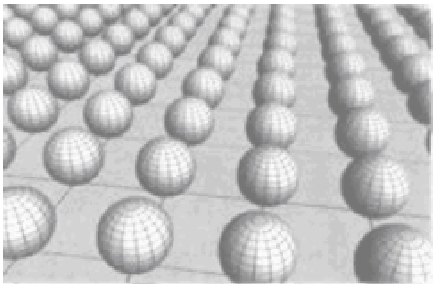

Figure 6. Calab-Yau space.

Now, this is quite a coincidence, because Greene is here exhibiting those beasties known as Calabi-Yau spaces — the additional spatial dimensions of M-theory, usually thought to be vanishingly small because we do not "see" them — a notion inherited from the pioneering work of Kaluza and Klein and often stated in passing — and quite in keeping with the dogma from physical theory (which we can trace directly to the pioneering work of Galileo and Newton) that the world we see is "obviously" four-dimensional (4D). But the world we "see" is manifestly *not* 4D, for then the world would be quite literally invisible. What we "see" are, in fact, a flux of colored surfaces — a fact which ought to be of interest to those who pursue Chern-Simons theory, which links gauge theory to M-theory by way of a bridge supplied by topology.

Minkowski clarifies the essential situation in his famous essay on relativity:

> We will try to visualize the state of things by the graphic method. Let x, y, z be rectangular co-ordinates for space, and let t denote time. The objects of our perception invariably include places and times in combin- ation. Nobody has ever noticed a place except at a time, or a time except at a place. […] The multiplicity of all thinkable x, y, z, t systems of values we will christen the world.

To paraphrase a bit, "the objects of our perception" invariably exhibit the truth of Wittgenstein's observations concerning how "a speck in the visual field, though it need not be red must have some color […]" Once one accepts that colors and sounds and so forth are simple physical entities which define manifolds, it is but a short step to "seeing" that color space fibers over visual space — and so with sounds and aural space, etc. On this view, a subspace of Calabi-Yau space looks like this:

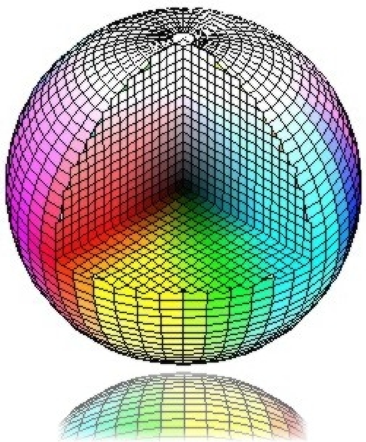

Figure 7. Color space

Conclusion

This shift of Gestalt will prove a jolt for many, running counter to a worldview that Galileo et al. inherited from Democritus and the atomists of classical antiquity. The shift is facilitated by the durable, reliable nature of the secondary qualities or properties — i.e., they are not going away and their behavior is open to inspection, test, prediction and falsification. To choose two instructive examples, consider that colors and sounds cannot reveal whether one is at rest or in a uniform state of motion. Likewise, the "red" of a red-shift alone cannot tell us whether the source is receding or in a gravitational field.

It seems, therefore, as though colors and sounds respect the symmetries embodied in both the special and general theories of relativity.

There are also a few simple physical facts which seem to have eluded many of our critics.

Given the evident symmetries and phase relations of colors and sounds, together with the apparent fact that their respective spaces fiber over their associated sensory spaces, one wonders, again, whether colors and sounds (along with the other secondary properties) might occupy the internal spaces of gauge theory and/or the additional spatial dimensions of M-theory.

Notice that this move addresses one of the most persistent complaints regarding M-theory, namely its failure to provide observable predictions. For on the view suggested here, we really do "see" the additional dimensions — they have simply been obscured by centuries of dogma.

Returning to the place where we began: Apropos the seminal work of Andras Pellionisz and Rudolpho Llinas, the Churchlands and others have argued for a "tensor network theory" of neural function. It seems as though the views of these authors comport well with those expressed above, especially in respect of:

1) The neural implementation of matrix operations on input vectors; and the view that
2) the cerebellum's job consists in "the systematic transformation of vectors in one neural hyperspace into vectors in another neural hyperspace"; and the notion that
3) "the tensor calculus emerges as the natural framework with which to address such matters," together with
4) the characterization of phenomenal properties as vectors; and
5) the physics and mathematics of the secondary properties.

We arrive at a picture wherein both "mental" perception and its "physical" substrate are mediated by one set of matrix-valued tensors which operate on sensory input vectors to yield both what we perceive and what we do by way of behavioral responses. (Or, as I like to say, neural form follows quantum function.) Given the fractal character of dendritic arborizations, we might expect to find this kind of self-similarity across temporal and spatial scales.

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