Aspects of Reference and Information

Daegene Song

ABSTRACT

The self-reference method has yielded some of the most important results in the study of mathematical logic and computation. The physical realization of the self-reference method has been said to be the self-observation of consciousness, which suggests a non-computable element (i.e., there exists a natural phenomenon that cannot be computed). However, the contradiction or paradox in self-reference may be solved by adopting cyclical time. Contrary to the familiar notion of time as a linear progression, the cyclical concept of time suggests that time circulates. In this cyclical time model (i.e., $t_0 \rightarrow t_1 \rightarrow \ldots t_N \rightarrow t_0$), the non-computable element in self-observation can be considered to be computable.

Key Words: Self-reference, self-observation, cyclical time

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Introduction

One of the central ideas in the study of logic has been the notion of self-reference (i.e., something that references itself) (Bolander, 2002). For instance, this particular approach has been used to establish some of the most important results in mathematical logic, including Gödel's incompleteness theorem, which showed that sufficiently complex formal axiomatic systems cannot be both consistent and complete (Gödel, 1931). Since this abolished the proposal to establish all mathematical theories on a firm ground of axioms and proofs, it can be said that it shows, loosely speaking, the imperfectness of mathematics. Unpredictability and randomness, which can be considered to show the imperfectness of physics at the fundamental level, were explored in quantum theory at the beginning of the 20th century (Peres, 1997). Therefore, it is plausible to speculate that the imperfectness of these two disciplines is linked together.

Another interesting and important area of study is related to the notion of consciousness, which has traditionally been approached from the perspective of neuroscience (Koch, 2004) and neural network models (Harvey, 1994; Russell *et al.*, 2002). One of the reasons why consciousness has been so interesting yet so puzzling is that it describes a natural phenomenon in which the observer is aware of him or herself. This is unique because observation is conducted in terms of the relative difference between the observer's reference frame and the object being observed. However, in the case of consciousness, the observer is observing his or her own reference frame. That is,

The observer observes him or herself *(1).*

Since the self-reference used in logic is a case that refers to itself, self-observation can be considered the physical realization of self-reference in logic. Therefore, it is possible to contemplate whether consciousness is related to the self-reference in logic and the imperfectness of physics, in particular quantum theory. Indeed, in (Song, 2007), it was shown that one can connect quantum theory with consciousness using the logic of self-reference.

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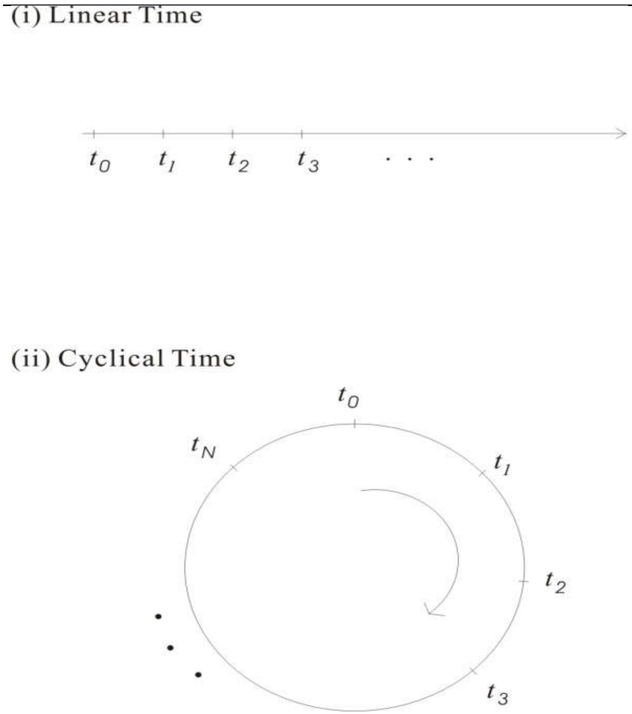

Figure 1. (i) Linear time: this corresponds to the traditional notion of time used in physics. That is, time evolves from t_0 to t_1, t_1 to t_2, and so on. (ii) Cyclical time: this is the case where time circulates. That is, at a certain t_N, where $N > 0$, it goes back to t_0.

In this paper, we wish to add another aspect to the analysis of consciousness and self-reference: time. In physics, where causality has traditionally been one of the central underlying principles, time proceeds in a linear direction. For example, it can be said that it proceeds from t_0 to t_1, t_1 to t_2, t_2 to t_3, and so on. However, it is also possible to consider a model where time circulates, proceeding from t_0 to t_1, t_1 to t_2, and so on, but at a certain time t_N, where $N > 0$, t_N proceeds back to t_0 (Figure 1); this is called cyclical time (Reynolds, 1994; Meyer, 2009). Although it may seem counter-intuitive, the concept of cyclical time is in fact observed in a certain culture (Boman, 1960). In this paper, we will discuss how the self-reference seen in consciousness, as shown in (1), can be considered cyclical time. Indeed, it will be argued that the non-computable aspect of consciousness, as seen in (Song, 2007) (which was based on linear time), can be computable in cyclical time.

One well-known and simple case that uses self-reference is the statement known as the liar's paradox, which is as follows:

This statement is false. (2).

This seemingly innocent looking statement is paradoxical in the sense that if the statement is true, then it implies that the statement of falsity is false, which implies that the statement must be true, but this contradicts the initial claim of falsity by the statement. If we instead assume the initial assumption to be false, this suggests that the statement is true, but this implication that it is true contradicts the statement's initial claim of falsity. Either way, there is a contradiction. Note that in the example of the liar's paradox, the statement is set in such a way that when it refers to itself as false, the statement runs into a contradiction. In fact, the contradictory aspect is hinted at in the infinitely recursive pattern that the statement possesses, as seen in Figure 2.

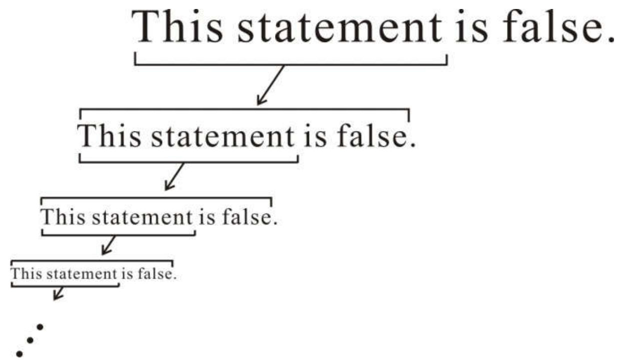

Figure 2. The Infinitely Recursive Aspect in the Liar's Paradox .

In the late 19th and early 20th centuries, the foundation of mathematics was being examined. In particular, Frege attempted to establish mathematics on a firm logical ground by constructing a formal system (Frege, 1903). However, Russell used self-reference to provide an example of a set that leads to a contradiction (Russell, 1902). The self-reference set, X, can be defined as a set of sets that are not members of themselves. Similar to the liar's paradox, this also leads to a contradiction: either the set X is a member of itself or not.

In 1936, Turing published a paper that laid out the foundation for modern-day computers (Turing, 1936). The proposed machine was to perform mechanical computation. Based on his theoretical invention, he was able to show that it was possible to construct a well-defined function that could not be computed, again using the method of self-reference. Another well-known result of self-reference is Gödel's incompleteness theorem in mathematical logic. At the beginning of the 20th century, Hilbert suggested that mathematics should be established on a solid ground of consistent axioms and proofs (Hilbert, 1900). In 1931, Gödel showed that in any sufficiently large system of axioms, one can construct a statement that can neither be proved nor disproved (Gödel, 1931), thereby showing that it was impossible to achieve Hilbert's proposal. Once again, the method of self-reference, similar to the liar's paradox, was central to Gödel's argument (Figure 3).

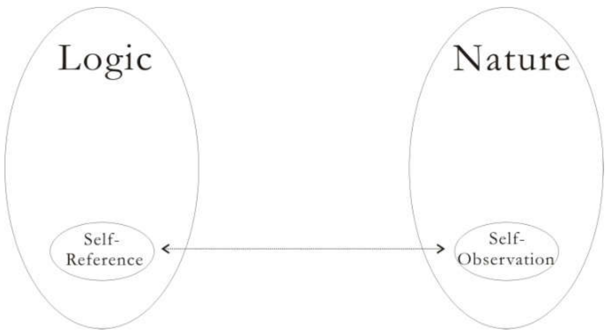

Figure 3. Self-reference and Logical Structures: In a system or a structure, the failure to establish consistency can result from the existence of contradictory self-references. This can be seen in Russell's paradox, Gödel's theorem, and consciousness in quantum physics.

Self-observation in Cyclical Time

In order to discuss consciousness in the cyclical time model, let us first review the non-computability in self-observation. In particular, we wish to discuss the computability of consciousness with regard to two aspects: First, can we represent consciousness in terms of a computing model? In order to develop a physical theory of certain natural phenomena, it is important to provide a reasonable assumption in order to guess the mathematical description of it. Based on this guess, one then attempts to find a pattern or a rule through the equation.

In (Song, 2007), it was argued that the mathematical description for consciousness can be assumed using the existing quantum theory by taking the observable as a reference frame for the observer, while the state vector can be used as a mathematical description of the object to be observed or measured. This assumption is based on the dilemma that quantum theory has faced since its birth a century ago, namely the measurement problem. It has been called a problem because the theory, or at least the standard Copenhagen interpretation, assumes a special status for the observing party. That is, the theory provides a subjective relation between the observing party and the object rather than provide an objective description of a physical system or phenomenon, which had been the case for centuries before. However, the subjectivity of quantum theory becomes advantageous in providing a mathematical description of consciousness because the theory provides the mathematics not only of the object but also of the observing party.

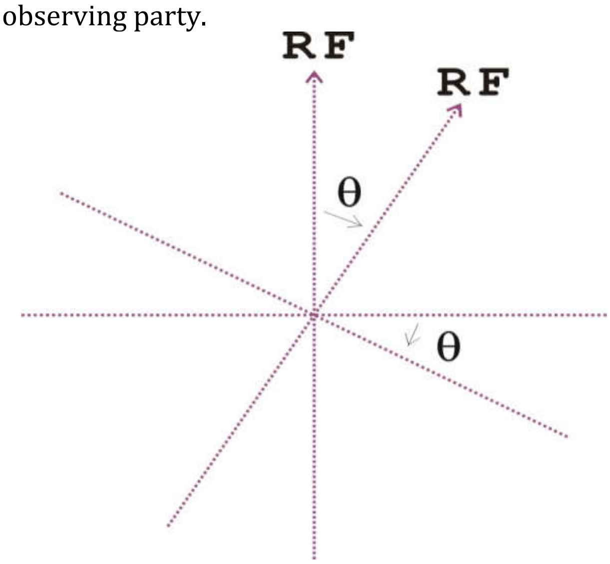

Figure 4. The usual transformation in quantum dynamics, where the state vectors and the observable (or the reference frame) are distinct, corresponds to the case in which the Schrödinger and Heisenberg pictures provide the same observational outcome. In the case of self-observation (i.e., when the observer's reference frame is equal to the object to be observed, as seen above), the symmetry is broken.

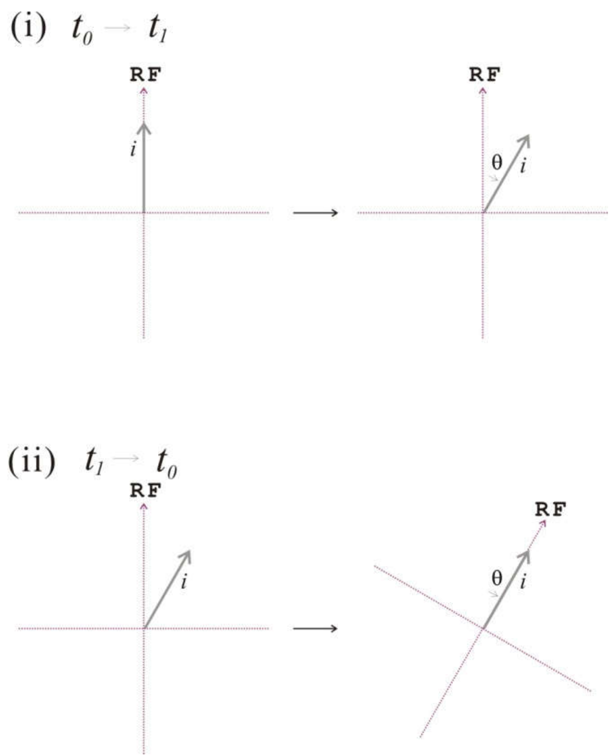

Figure 5. Computation in cyclical time: with a temporary vector i, the first step (i) is the rotation of i by θ from $t_0 \to t_1$. The next operation (ii) is the rotation of the reference frame by the same amount, such that it is in the same position as i with $t_1 \to t_0$. This procedure in cyclical time is equivalent to self-observation in linear time.

The second aspect that we wish to discuss is computability and symmetry. We are concerned with symmetry in computation with regards to the computability of a given task (e.g., the symmetry present between the reader and the tape in Turing machines; i.e., the head moving a step to the left or right is equivalent to the tape moving a step to the right or left, respectively). This seemingly obvious property can also be found in quantum computation with the equivalence between the Schrödinger and Heisenberg pictures, which happens in the measurement because of the separation between the observing party and the object being observed. In this paper, when the computability of consciousness is examined, we will consider whether the given computation obeys this symmetry.

Using the notation of the Bloch sphere (see (Nielsen *et al.*, 2000) for a review) the basic unit of quantum information, a qubit, can be represented as a unit vector, which is assumed to be pointing along the *z*-axis. In a similar fashion, the coordinate or the reference frame of the observing party is represented with a unit vector, which also initially points along the *z*-axis. In the Schrödinger picture, the unitary transformation is applied to rotate the qubit by θ on the *x-z* plane. In the Heisenberg picture, it is the coordinate vector that is rotated counterclockwise by the same amount, and the outputs are equivalent in both pictures. However, the non-computable part (i.e., when the symmetry is broken) appears when the object that is being observed is the reference frame itself. That is, in the Schrödinger picture, the vector would be rotated by θ, while in the Heisenberg picture, the same vector would be rotated counterclockwise by the same amount, and the two are not generally equivalent. In (Song, 2007), it was shown that this inequivalence necessarily implies the non-computability of self-observation or consciousness (Figure 4).

We now wish to discuss how the non-computable self-observation (i.e., the physical realization of self-reference in logic) can be considered to be computable (i.e., the symmetry is preserved in cyclical time). As discussed earlier, cyclical time corresponds to a time transition of $t_0 \rightarrow t_1 \rightarrow \cdots \rightarrow t_N \rightarrow t_0$, etc. In this case, we will consider a simple case of cyclical time in which $N = 1$. As shown in Figure 5, we wish to introduce a temporary vector $i = (0,0,1)_i$ that points in the same direction as the reference frame. The procedure of computation in cyclical time has two steps. In the first step, where $t_0 \rightarrow t_1$, the vector i is rotated by θ as follows: $(0,0,1)_i \rightarrow (\sin\theta, 0, \cos\theta)_i$ (Figure 5 (i)). This is commonly seen in ordinary computation and is certainly achievable in both pictures. In the second step of cyclical time, where $t_1 \rightarrow t_0$, we consider a rotation of the reference frame by θ such that it equates itself with the vector i, $(0,0,1)_{RF} \rightarrow (\sin\theta, 0, \cos\theta)_{RF}$ (Figure 5 (ii)), which is symmetric in the rotation of the Schrödinger and Heisenberg transformations. The final result is equivalent to the self-observation of the reference frame by θ, as shown in Figure 4.

Therefore, it can be seen that the formerly non-computable procedure, which did not preserve the symmetry between the Schrödinger and Heisenberg pictures (Figure 4), now becomes computable because the procedures preserve the symmetry when the cyclical time model is adopted. Earlier, self-observation was introduced in (1), which was argued to be a physical version of self-reference in logic (Figure 3). The statement can now be understood in the cyclical time model as follows:

1. The observer observes something
2. The something was the observer

That is, from t_0 to t_1, the observer is observing something. In cyclical time from t_1 to t_0, the something that was observed refers back in time to be the observer (Figure 6).

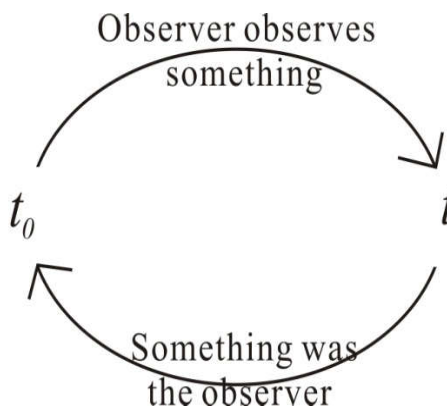

Figure 6. Self-observation in cyclical time: From t_0 to t_1, the observer observes something. In the cyclical time of t_1 to t_0, the something that was observed refers back in time to be the observer.

Remarks

As discussed, self-reference, which references itself, is one of the central aspects in the inconsistency or imperfection of logical structures. In particular, self-observation or consciousness has been argued to be a physical example of self-reference. In this paper, we have discussed how self-observation, which exhibits non-computability in a manner similar to paradoxes in logic, may be computable in cyclical time. That is, what is non-computable in linear time can be computable, and obey symmetry, in cyclical time. In (Song, 2014; 2015), self-observation, as shown in Figure 3, was associated with the observer's free will as a nondeterministic computation (i.e., the time reversal of irreversible computation). The conclusion, relating free will in self-observation with cyclical time (as discussed in this paper), appears to be consistent with the philosophical discussion in which a similar proposal was made from a different angle (Arendt, 1978; Deutscher, 2007).

Acknowledgments

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